LAND SURVEYING.

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A TREATISE

ON

LAND SURVEYING

IN THEORY AND PRACTICE.

GIVING THE BEST METHODS OF SURVEYING AND LEVELLING FOR STATISTICAL, ESTATE, AND ENGINEERING PURPOSES:

together with

FULL EXPLANATIONS OF THE CONSTRUCTION, ADJUSTMENT, AND USE OF THEODOLITES, LEVELS, AND OTHER INSTRUMENTS REQUIRED IN THE FIELD AND OFFICE WORK OF SURVEYING AND LEVELLING.

FOR THE USE OF

ENGINEERS, SURVEYORS, AND STUDENTS IN COLLEGES AND SCHOOLS.

BY

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WITH NUMEROUS ILLUSTRATIONS.

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PREFACE.

In the following Treatise the Author has aimed at bringing before the reader the best methods for practice in the art of Land Surveying, and the most convenient formula for computations in working out the required results. As the subject embraces surveys for Statistical, Estate, Colonial and Engineering purposes, it has been a leading object to attain practical excellence in the *field* and *office* work required of Surveyors and Engineers of skill and ability. This object will be best secured by holding *practice* subject to *theory*, properly applied, so that sufficient verifications may be readily had and the degree of accuracy easily ascertained. The operations requiring attention for securing these essentials of a good survey are concisely, yet fully, described, without specially directing the attention of the reader to a different faulty practice, in order to condemn it, except in some cases where practice has been apparently, but not properly, based on theory. In describing surveys in which theory has been much disregarded, and the instruments used not suitable for

the service, the Author has drawn attention to, and thrown out suggestions for, improvements in the means and on the modes for making these surveys. Although Land Surveying is by no means a new subject, still, until a comparatively recent period, the art was not skilfully practised. Even at the present time authors of works on the subject do not sufficiently realise the existence and extent of the obstacles the surveyor encounters in the field—particularly in extensive surveys.

In a work on the subject much absolute novelty cannot be expected. The Author believes, however, that the present work contains much that is new to the public, whilst he hopes the treatment of parts not altogether new will be appreciated by readers desirous of acquiring skill and ability in the designing and making of really good surveys. He would follow the example of others, and lay claim, in this place, to the new matters, and the improvements on operations not altogether new which are described in many places throughout the work, if such information were of much practical importance to the reader, or if it would add to the value of the work. As he is, however, not quite certain that these desirable objects would be served to any great extent by stating in a preface the particulars of his special claims, he is satisfied to allow the text to be his index.

The work is divided into two parts. Part I.

relates to the Field Work, and Part II. to the Office Work and Instruments used in Surveying, &c. The Author's arrangement of these parts enabled him to preserve the continuity of the descriptions in the body of the work, by collecting the examples of computations from the formula dispersed throughout the descriptions, in a separate Chapter. It is hoped this separation of the *descriptions* and *example computations* will be found, not only by engineering students in colleges and schools, but also by professional and private gentlemen, to be advantageous for the study of the subject. And it is also hoped that the numerous woodcut and copper-plate illustrations will be found to greatly assist the reader in fully comprehending some parts requiring particular attention.

Belfast: *July* 6, 1869

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ON

LAND SURVEYING.

PART I.

CHAPTER I.

GENERAL REMARKS, PRELIMINARY TRAINING, ETC.

When it is required to ascertain the area of a given plot of ground, or when it is required to obtain, to a diminished scale, an accurate representation on paper of the detail and boundary lines of a parcel, plot, or district, a survey of the parcel or district is necessary, in order to furnish data for the above purposes. The survey may be made to furnish data for laying down on paper a map or plan only. It may also be made to furnish data for ascertaining the area, and laying down an accurate map.

The area of any defined surface is the number of square links, square feet, perches, roods, acres, square miles, &c., contained within its enclosing or boundary lines. In Land Surveying for Statistical, Estate, and other like purposes, the surface, the area required to be ascertained, is that on a horizontal plane, to which the position of the bounding and other detail lines shall be referred by vertical lines. When the district is very extensive, the area on the horizontal plane would not be strictly correct. In these cases the area should be computed as for a spherical surface, at a given distance

from the centre of the earth (usually the level of the sea), to which the vertical lines will correctly refer the position of the detail. In hydrographical surveys, and surveys for a map only, the position of the detail is also referred to the horizontal plane by vertical lines. The data obtained in this manner will have reference to the position of the detail referred to the horizontal plane.

By 'detail' is meant objects or outlines, natural or artificial, such as rivers, water margins, beach marks, lakes, seas, and precipices, &c.; walls, houses, fences, roads, canals, and other public ways, &c.

A map, which is a general representation of surface and other features, is an accurate representation, laid down to a diminished scale on a plane surface, such as a sheet of paper. The corresponding surface is in the horizontal plane, to which, in the survey, the detail shall be referred.

In surveying operations two classes of instruments may be used—namely, instruments for measuring distances, and instruments for measuring angles. Chains, or other measures of equal parts, are generally used for measuring the former, and the theodolite, or other suitable angular instruments, are used for measuring the latter. In laying down on paper a correct representation of the detail surveyed with these instruments, plotting scales, or measures of equal parts to the diminished scale, are used for laying down the linear measurements; and protractors, or graduated circles, for laying down the angles, or angular measurements.

On the skilful use of the Field Instruments, and the geometrical construction adopted for the survey, will mainly depend its excellence. A careless or unskilful use of the instruments, or an ill-conditioned geometrical

construction, will render the survey one of an inferior character.

On the skilful use of the Office Instruments, and the accurate delineation of the plotted detail, together with accuracy in the field work, will depend the character of the office work and the accuracy of the map or plan.

The Surveying Chain.

The surveying chain is a linear measure of equal parts. In these countries the chain is generally made of round iron wire, in equal links, or lengths, to the number of 100. These links are connected by circular or oval rings. The extreme ends are furnished with swivels and open handles of brass or iron. The chain of 100 links is usually divided into 10 equal parts, of 10 links each, by brass pendants, the outer edges of which have prongs, to indicate the number of links from the nearest end of the chain to the brass pendant. The pendant at the middle, or 50 links, of the chain is rounded on its outer edge. The best chains of this manufacture are those which are light, yield least under a strain of about 30 lbs., and recover their original lengths on removing the straining weight. Chains are made of various lengths, depending on the object and field of the survey.

In Land Surveying for Statistical, Estate, and like purposes, a chain of 66 feet in length is very generally preferred. For engineering and some other purposes, a chain of 100 feet in length is more convenient than a chain of 66 feet, in which a link is less than one foot in length.

On the Continent the linear measure most approved and in most general use is manufactured of a single

piece of steel tape, furnished with T-handles at the ends. On some measures of this make the secondary divisions are marked in the process of rolling the tape, and on others the secondary divisions are marked with brass discs, riveted to the tape. These measuring instruments are usually 20 to 30 metres in length. In clear open country, or in the hands of a careful surveyor, these measuring instruments are decidedly superior to the chain constructed of many separate pieces connected by rings. The latter is liable to stretch, by the partial opening of the numerous rings, whenever the chain is subjected to an over strain. This is a circumstance of frequent occurrence in the field.

THE CORRECT LENGTH OF THE CHAIN.

Iron, of which chains are manufactured, expands with increase of temperature. Hence a chain has a 'standard length' at a 'standard temperature.'* Whenever the temperature changes, a corresponding correction should be made on the length of the chain, to bring it to the *standard* length. If a chain be applied to the 'standard,' and, when drawn by a strain of from three to five pounds, its length be made equal to the 'standard' length, and the temperature of the chain be noted, the chain may be afterwards used for laying down a temporary standard in a distant locality. For this purpose the chain should be undisturbed in the meantime. In laying down a standard with the chain, regard must be had to the proper strain

* The standard measure of length, for surveying purposes, is a multiple of the standard yard kept in the Exchequer Office, Westminster. Under mean atmospheric pressure, the standard temperature is 62° Fahr.

and temperature. For this purpose let l be the standard length of the chain, t the corresponding temperature in degrees (Fahr.), and t' the temperature in degrees (Fahr.) when the chain is extended for laying down the 'standard.' Then $\frac{\pm t \mp t'}{153,000} \times \frac{1}{l}$ will be the error between the chain and standard lengths. It is desirable to have always ready for use a standard measure, such as a well-constructed chain, or a set of glass rods, in order to restore the standard, should the 'standard' marks on permanent objects be obliterated or otherwise destroyed.

The permanent objects on which the standard may be laid down for daily use should be easily accessible, such as kerb-stones, flagged pavements, the cut-stone coping of a bridge, or other like surfaces. The standard length should be finely marked by well-defined transverse cuts, made accurately at the standard distance apart. The chain for use in the field should be applied to this standard, before and after each day's field work, to ascertain its length. If the length of the chain exceeds the standard length by one or more rings, these should be removed at or near the ends, and the error remaining, if any, noted in the field-book. This error should be entered to the work surveyed with the chain in that condition. If from any cause the chain be broken in the field, the circumstance should be noted in the field-book. The chain should not be again used in the field, after repairs, before its readjustment to the 'standard' length.

For surveys of considerable extent, it is desirable that a 'standard' be laid down in the immediate neighbourhood of the survey operations. For this purpose a suitable surface may not be available. In this case the surveyor should select convenient ground, having an even

surface, into which two stakes should be driven, at about a chain's length apart, and saw-cut at the surface of the ground. On the saw-cut surfaces of the heads of these stakes the standard length should be marked as already described. In applying the chain to the 'standard,' care should be taken to free it of all dirt and ringlocks, which would interfere with its measured length.

Measuring Horizontal Distances.

The accurate admeasurement to the horizontal of a straight line not in the horizontal is essential to a correct survey. To acquire skill and ability in this operation is, therefore, very important indeed.

For the purpose of finding the horizontal distance in such cases either of two modes may be followed. In the one the chain only is used; in the other, in addition to the chain, an angular instrument is used.

Horizontal Distances—With the Chain.

In addition to the chain, the survey party should be provided with ten arrows, a conical plummet, a small spade, and several ranging-rods, or flag poles. The arrows are used for marking on the ground the measured chain lengths; the plummet is used for referring the chain length in the horizontal to the inclined surface; the spade is used for making pickets, piles, slashing thickets, and passing the chain through hedge-rows. The ranging-rods are used as field objects, to facilitate the accurate tracing of straight lines, and the surveyor's other operations in the field.

In the survey operations, the first and very important preliminary matters to be attended to are—to judge

when the chain is held in the horizontal plane, and when it is drawn out to its 'standard' length. For the purpose of training the eye and hand to determine accurately these conditions the following practice is almost indispensable :—

On ground sloping at about 1 in 14, wands should be set up at about ten links apart, and cross-pieces in the horizontal plane attached to them, so that the chain, when attached to the higher ground, and extended on the cross-pieces, may be throughout in the horizontal plane ; the end of the chain on the higher ground should be securely attached to it by arrows or otherwise. The free end of the chain should be held to the line of the cross-pieces on the wands, and the chain fully drawn out When the chain is so held and extended, its length should be referred to the surface of the ground, by letting the plummet fall free from under the free handle to mark on the ground with its conical point the correct chain length in the horizontal referred to the inclined surface. This should be repeated several times to verify the marked surface point, or chain length, and the eye trained to judge the horizontal when placed at the higher or lower station, the cross-pieces should be removed from all the wands except the one next the lower station. The free handle should be now held to the former level, as shown by the remaining cross-piece, and the chain drawn out with sufficient force to make the plummet in its free fall from under the handle of the chain hit the previously determined correct chain length point on the surface. This practice should be continued until the hand estimates accurately the necessary force, and the eye judges correctly when the chain is held in the horizontal. All the wands should be now removed, and the previous training tested by drawing out

the chain in the horizontal, and referring its correct length to the inclined surface as before. Like practical training for other inclinations should be gone through until the eye and hand be sufficiently educated for their respective duties in judging when the chain is in the horizontal, and strained so as to measure the correct length.*

The eye once trained will judge the horizontal for all practical purposes; but the hand must be frequently trained to measure the strain to be applied to each chain, and to the same chain at different times during its use. The practical importance of this training cannot be over estimated. The survey party who may have acquired sufficient skill and ability in laying down on the ground a chain length in the horizontal plane will, by attending to the following, find little difficulty in measuring accurately the horizontal length of a straight line.

Measurement of a given Straight Line.

In proceeding to measure a given line, the surveyor's assistant should draw out the chain approximately in the line to be measured. The surveyor, or 'follower,' who determines the trace of the line forward, should at the same time place the eye, with which he determines the trace, in the vertical plane of the line, and holding the handle of the chain directly under the tracing eye,

* The chain, when supported at the ends only, lies along a catenary curve. In this position the direct length of the chain is less than the curved or true length. The chain composed partly of numerous rings, cold butt-jointed, when subjected to a sufficient tensile strain, will elongate, by the opening of the ring-joints, to an amount, or length, equal to the difference between the direct and curved length, so that the direct length of the chain under strain may be the correct length.

he should glance along the chain held tight in the straight line, by which he will perceive if the chain lie in the trace of the given line. If the chain be not in the trace or vertical plane of the given line, the surveyor should direct the 'leader,' by signal or otherwise, to change his position until the chain be accurately adjusted in the vertical plane of the given line. The surveyor should then pass a few waves through the chain, with a slack hand, to free it from dirt and ringlocks, and holding the handle of the chain accurately to the end, or arrow, as the case may be, direct the leader to keep the chain fully drawn out, and to mark accurately the chain length with an arrow. This should be done on soft ground, or till, by driving the arrow vertically into it: on hard ground, such as rock, roads, &c., the chain length should be marked by scribing two lines with the point of an arrow from the proper point, thus $\wedge$, at which mark an arrow should be left for the follower. If the ground be not horizontal in the direction of the line, the chain length in the horizontal should be determined on the ground as above described. The point should be marked with an arrow. Whenever the follower may be at the lower point the chain length cannot be laid down, for certain, at one operation. In this case the leader should drive the arrow into the ground as near as may be at horizontal chain length, and hold the handle of the chain accurately to or over it, as the follower may require, whilst the latter should determine the chain length in the horizontal, and refer it to the ground as already described. The difference between the point so determined on the surface and that marking the forward end of the last chain length laid down, if any, should be accurately measured and applied ($\pm$) to the forward arrow-point, to accurately determine the point for the chain length. To

this point the arrow should be removed before again adjusting the chain in the line. The leader should be careful not to leave any arrow after him except those marking the chain lengths. These the follower should take with him as he completes each chain length in the measurement of the line. The surveyor should not draw the arrow at his end [of the chain length being laid down], before his assistant has finally driven the arrow into the ground, or otherwise marked the point for the forward end of the chain length.

Skilful surveyors, in order to economise time, in measuring on long and uniformly inclined planes, on which there may not be detail, measure a chain length to the horizontal and a corresponding chain length on the inclined plane, and so determines the error due to a chain length on the incline. The surface measure may be that taken for the remainder of the incline. The correction for inclination should be made at the end of the incline by changing the position of the arrow, forward, a distance equal to the ascertained error, on one chain length multiplied into the number of chain lengths measured to the surface. This expedient to economise time should not be adopted, even by skilful surveyors, if a high degree of accuracy is an object, since any error fallen into in the determinations on the one chain length increases with the distance. When each chain length is measured to the horizontal, the errors compensate, to an unascertainable amount, and hence the greater attainable accuracy.

On grounds sloping so that the chain cannot be held to the horizontal at its end, the leader should take up the chain, in downhill work, at a point which he can hold to the horizontal, and refer that point in the horizontal to the inclined surface, as already described. The

point on the chain should be adjusted to the corresponding point on the surface, and in this position handed over to the follower. The remaining part, or parts, of the chain should be held to the horizontal, and their lengths referred to the inclined surface, until the whole length of the chain be so referred: the leader should mark with an arrow the end of the chain length. In uphill work a similar process may be followed, with this difference, that the leader should shift his point on the surface, until the plummet, now used by the follower, marks the point on the surface last determined; then will the leader's point be accurately found. The chain should not be disturbed from the point on the surface to which it is held by the leader, until the follower takes it up for the purpose of laying down the next forward part of the chain length.

The operation of laying down a chain length to the horizontal on the level, and in *up* hill or *down* hill work, described above, should be repeated, as may be required, for each successive chain length until the whole distance be measured.

If the measured distance be less than ten chains, the number of arrows taken up by the follower will be the number of chain lengths laid down, to which should be added the links (if any) or fraction of a chain, to find the number for entry in the field-book—the expression in chains and links for the whole measured distance.

If the distance exceeds ten (or any multiple of ten) chains, the arrows must be passed from the follower to the leader at the forward end of each tenth chain laid down, which should be marked with a picket, thus $\xrightarrow{\text{Y}}$, so that the acute angle may accurately mark the tenth arrow point. Every change of arrows should be entered on the field-book, so that the whole length of the

measured line may be expressed by adding the number of arrows less than ten, and the links (if any) to the number for ultimate change of arrows, thus 30 + 5 + 0·84 will read and should be entered 35·84. It is customary and advisable to enter the measurement in links (3584).

It has been found, on the Ordnance (detail) Survey of these countries, that fair average surveyors, making large progress in open country, have measured their lines to the horizontal, in the manner described above, with the maximum error not exceeding 1 in 1,000. Like surveyors, in close detail and stiff hilly country, making large progress for the class of work, have brought the maximum errors within 1½ in 1,000. Superior surveyors have brought the chain measurement of the sides of triangles, in undulating and hilly districts, to within 1 to 2 links in the mile of the computed distances.*

Finding Horizontal Distances with the Chain and an Angular Instrument.

Let the distance to be obtained be that of a straight line between two distant points in a hilly country.

For the purpose of the survey, two separate parties proceed—the one previous to the other—along the surface trace of the line. The surveyor first proceeds to make the measurements with the chain to the surface, in the trace of the vertical plane, passing through the given extreme points. The measurements may be commenced at either end of the line, and should be continued to the other end. In making the detail and reference line measurements, the surveyor should mark

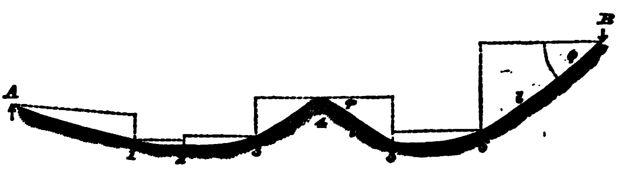
* The distances of the sides of the triangles were computed in the trigonometrical survey.

on the ground each change of inclination—such as 1, 2, 3, &c. (fig. 1)—by a picket and pile. The entries of the measurements for these pickets should be distinguished on the field-book by the initial L.P. (level point). After the survey of the line shall be completed, a copy should be furnished to the party for levelling the line.

The theodolite, or 'level party,' when furnished with a copy of the survey of 'detail' and 'level points,' is enabled to follow the trace of the line, and make the angular measurements (ϕ) for inclination of the several planes selected by the surveyor (such as A—1, 1—2, 2—3, &c.). From these data the horizontal distance may be computed for each plane selected, and the sum taken to find the horizontal distance for the whole line (A B).

If the surface be flat, or a plane, the surface trace of the line, viewed on a vertical plane, will be a straight line, horizontal or inclined, according as the surface is horizontal or inclined in the direction of the line. If, however, the surface be concave or convex, as in hollows and on hills, then its trace on the vertical plane will be curves of a corresponding character. These features

Fig. 1.



may be illustrated by the sketch (fig. 1), which shows a surface trace on a vertical plane.

In the sketch A B are the distant points; A, 1, 2, 3, &c., B are level points (L.P.) in the surface trace.

The horizontal distances corresponding to each incline may be computed by substituting values in the formula—

$$r \cos \phi \, l = h$$

In this formula r represents radius, ϕ the angle of inclination with the horizontal, l surface length of the incline (4—5), and h the horizontal distance corresponding to l. If $r = 1$, $\cos \phi$ may be taken from a table of natural sines and cosines, and the computations made without the aid of logarithms. The horizontal distance corresponding to each plane should be found, and the horizontal distance for each L.P. obtained by taking the sum of those distances from the first computed distance to that terminating in the point inclusive. These reduced distances should be entered, in red, to their respective L.Ps. on the surveyor's copy of the survey. The copy, so corrected for each level point and the whole line, should be returned to the surveyor, that he may transfer the entries in red to the original survey of the line.

It may be observed that the surface measure between consecutive level points may be greater, and can never be less, than the distance in a plane passing through these points. Hence errors due to the deviation of the surface from the levelled incline—*i.e.* the incline selected by the surveyor must *always* accumulate. And as these cases are of frequent occurrence, the computed must be greater than the actual horizontal distance by an unascertainable amount. With a view to reduce the amount of calculation required for making the reductions, which is very considerable, the vertical limbs of some theodolites are graduated to give the error in links per chain length, of 100 links, in the inclined surface. The saving in

time thus effected is obtained at a great sacrifice of accuracy, and hence the adoption of the expedient was ill advised.

Besides the great labour and chance of arithmetical error in finding the reduced horizontal distances for the whole line and the several level points, much labour in computations is further required to reduce the surface measure of every detail and reference point in the line to the corresponding horizontal distance; so that not one original measurement is allowed to stand without correction. From this it will be seen that the map or plan and computation for areas cannot be had from original but only from secondary documents, which may contain errors of calculation superadded to errors in booking and making the original measurements, and these are without any convenient means of correction.

There is another very serious objection to this method, as it is desirable that the measurements first entered on the field-book should be as few as may be necessary to give the data complete, and that each entry for computation or laying down a map on paper be an original entry.

From these considerations, and those already stated, it is obvious that for all practical purposes the measurements to the horizontal with the chain are preferable to the measurements to the surface with the chain, and the reductions of this measure to the horizontal by combining it in computations with vertical angles measured with a theodolite.

In the early part of the Ordnance Survey of Ireland the method last described was adopted for finding the horizontal distance in connection with the survey of detail. Subsequently, and on the revision of the districts so surveyed, the horizontal distances were obtained

by measurements to the horizontal with the chain. In every case the measurements made according to the latter practice were found to be more accurate than the computed results in the method first adopted. Some lines in the original survey, which had the reduced horizontal distance as much as about 30 links greater than the correct distance found in the trigonometrical survey, were, on the revision survey, measured to the horizontal with the chain, and brought to within three links of the computed distance.

CHAPTER II.

SURVEYING FOR DETAIL WITH THE CHAIN—GEOMETRICAL FIGURE.

In the foregoing Chapter the general practical survey operations are treated with respect to the simple measurement, or determination, of the horizontal distance of a given straight line. In this and the following Chapter, relating to the survey of a district, the geometrical construction, or figure, for selection by the surveyor, and the expedients to be adopted to work out the construction in practice, will come more immediately under consideration.

Surveying with the Chain.

For a district of moderate extent, in fair open country, surveys of detail for maps and areas may be sometimes advantageously made with the chain, without the aid of other instruments. Besides the instruments, implements, &c., already described, the surveyor should be provided with an offset staff, for measuring perpendiculars and other short distances referred to the construction, geometrical, or reference line. The offset staff may be made of any light material, such as fir, poplar, &c. It should be ten links in length (ferrule-shod at the ends), and subdivided by suitable marks to single links. The form is not very material, but a round staff is that most generally preferred.

The district to be surveyed should be examined by the surveyor as to form and physical features, before selecting or determining the geometric figure for the purposes of the *survey*. In the determination of this figure, and its subdivisions into triangles, two leading considerations must be kept in view—the well-conditioned character of the geometric figure, and facilities for making accurate measurements of its lines. The whole of the district for survey should be included in the geometric figure, the external lines of which will form a polygon. In selecting the polygon, the surveyor should have regard to the number of sides, since the fewer the number of sides the less the chance of error.

For the subdivision of the geometric figure or polygon, straight lines should be selected to intersect in one or more internal points, and terminate in the angular points of the polygon. In many cases it may be desirable and advisable to select a line from one angular point to another angular point of the polygon. The internal points in which the selected lines from the angular points intersect may be connected by straight lines, which will form an internal polygon or a triangle. If the internal figure be a triangle, the subdivision will be complete; but if it be a polygon, the subdivision must be continued by lines from the internal angular points to other internal points, and the reduction of the number of sides of each successive internal polygon be continued, as above, until the subdivision be complete. In this subdivision the number of triangles should be as few and as nearly equilateral as may be, due regard being had to the facilities for tracing and measuring the sides of the triangles.

An important object to be obtained by the triangulation is to accurately ascertain by measurements the

position of the angular points of the several triangles. Every point sufficiently determined will be a common point of intersection of at least three lines. Points determined by the intersection of two lines only, are not determined by more than one set of independent measurements. These points are, therefore, called 'hanging points.'

For the purpose of ascertaining the position of internal detail, the great triangles, which form the primary triangulation, should be subdivided by lines referred to their sides or to one another, so as to preserve in the subdivision, or secondary triangulation for detail, the features of well-conditioned triangles. The *split line*, which should be run in every independent detail triangle, is an important line in this subdivision. This line should have one end in an angular point of the triangle, and the other in the opposite side, which it should intersect as near as may be at right angles. The point in which a subdividing line intersects the side of a triangle or another subdividing line is called the reference point of the line terminating in it. Every such point should be common to two or more such lines if practicable. Hence from the reference point of the split line other subdividing lines should be run to intersect the other sides of the triangle. This subdivision by triangulation should be continued to the extent required for the purpose of the survey. For the secondary, as for the primary triangulation, the surveyor should select the reference lines, so that the detail may be surveyed on them with the fewest measurements, the least practicable length of lines, and the greatest convenience for making the measurements.

The lines of the primary triangulation are sometimes called main construction lines, or main lines, and those

of the secondary triangulation detail lines. The distinction is not very intelligible, since the main lines are used for the measurement of detail, and some of the secondary or subdividing lines (detail lines) are run for the purpose of preserving the features of well-conditioned triangles —a good construction. There is, however, no essential difference between the geometric lines of the survey; they are alike selected not only for the purpose of preserving a good construction, but also for the survey of detail. On these lines are placed the reference points not only for other subdividing lines, but also for the perpendiculars and other lines to detail. All the geometric lines of the survey may therefore be called *reference lines*. For convenience and perspicuity the reference lines may be divided into two classes, *primary* and *secondary*, to distinguish the lines of the primary from those of the secondary triangulation.

A correct survey cannot be made if, from any cause, the reference points surveyed in the same line be not accurately in the straight line. Hence the necessity for ranging or tracing accurately all reference lines, and making the measurements, as already explained, in the line so ranged. A point is known to be in the line when from it straight, slender, vertical objects, at two or more points in the line, appear to coincide, or when a straight and slender object placed vertically on the point is seen from a known point in the line to coincide with a straight, slender, vertical object on another known point in the line. Therefore lines which may be seen from end to end are easily traced at intermediate points. Such lines, if of moderate length in open country, may be traced by the eye with sufficient accuracy for most survey purposes.

TRACING OR RANGING STRAIGHT LINES.

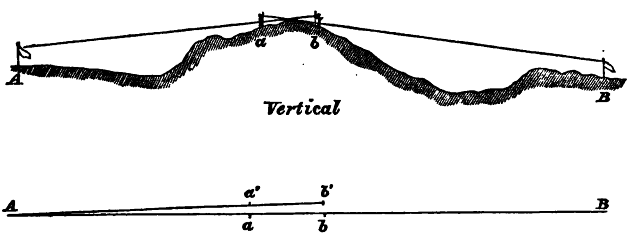
The tracing operation for lines of moderate length may be conducted as follows:—

When the whole of the line is visible from one end, the surveyor should send his assistant forward with a flag-pole or ranging-rod to a suitable point approximately in the line. At this point the assistant should hold the ranging-rod vertical and exposed to the surveyor, at the end point of the line, that the latter may observe if it covers or coincides with the flag-pole, or other object already placed vertical on the distant end of the line. If this coincidence be not obtained on the first setting up of the ranging-rod, it must be moved laterally, according to signal, until the flag-pole and ranging-rod coincide, then the latter will be placed on a point accurately in the line. The point so determined should be 'picketed' and 'piled' to mark it fully and render it conspicuous. Other points may be determined in the line in like manner; and, for greater convenience, the surveyor may continue the trace from one of these intermediate points. Ranging-rods are known to be vertical when the plummet-lines at the extremities of two rectangular diameters are parallel to the axis of the rod.

'When the whole of the line is not visible from one end,' the surveyor should place himself at a point in the line produced from which may be had a view of the flag-poles at the ends of the line. The flag-pole nearest the surveyor may be in this case regarded as an intermediate point. Other intermediate points may be determined from the surveyor's point in such position, that from the points so determined the surveyor may complete the trace of the line.

If there be no convenient point in the line produced commanding a view of the whole line to be traced, intermediate points must be determined in a different manner. For the purpose of the determination of intermediate points in the line so circumstanced, it is essential that either end should be visible from at least two intermediate points. The intermediate points for the determination should be sufficiently far apart to enable the surveyor to fix these accurately in the line. The determination of the intermediate points in the line will be easily understood by an inspection of the diagrams.

Fig. 2.



The first diagram shows a trace of the line $A\ B$ on a vertical plane. The positions of the surveyor and his assistants are shown at $a\ b$. The second diagram shows a plan of the line $A\ B$. For the purpose of the determination, the surveyor should take up a position at a', approximately in the line $A\ B$, so that B may be visible from a'. The surveyor should place a ranging-rod vertically at a', and the assistant should set up another vertically at b', in the line $A\ a'$ produced. The surveyor should observe if B be a point in the line of the ranging-rods at $a'\ b'$ produced. If B be not in the line $a'\ b'$ produced, the surveyor should take up a new position a

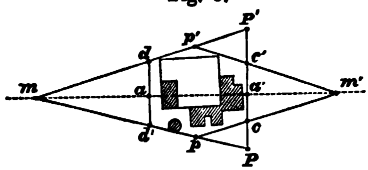
on the B side of a', and the assistant a new position b in the line $A\,a$ produced. If B be now observed to be a point in the line $a\,b$ produced, both a and b will be intermediate points in $A\,B$. From these points the line may be traced to its ends, as in the case first described. If it be found impracticable to trace the geometric lines of the survey by some one of the expedients described above, a modification of the figure becomes necessary. From this it will be seen that the angular points of the triangulation should be so selected as to facilitate the tracing of the reference lines terminating in it.

The detail trace of a reference line may be interrupted by buildings, trees, or other like obstructions at an intermediate point. In these cases the surveyor should trace the lines from the ends of the line to the opposite sides of the obstruction. If this be inconvenient he should set out equal perpendiculars, or parallel lines, at points in the reference line a considerable distance apart, so that a line traced through the remote ends of these equal lines may be parallel to the reference line, and clear of the obstructions. The parallel line may be traced past the obstruction, and from points in it, a considerable distance apart; other equal perpendiculars to those first measured should be erected and measured on the line side, so that a line traced through their remote extremities may coincide with the trace of the reference line, by which the latter may be traced backwards to the obstructing object, and forward as if no such obstruction existed. If the perpendiculars, or parallel lines, be not correctly traced and accurately measured, the trace on the forward side of the obstruction will not be made with the required accuracy for a correct survey.

The line may be traced differently as follows:—From

a point m (fig. 3) in the reference line $m\,m'$, sufficiently remote from the obstruction to give the direction of the line with accuracy, lines should be traced on either side of the obstruction, such as $m\,d\,p$ $m\,d'\,p'$, and the

Fig. 3.



$d\,a\,d'$ measured, a being a point in the reference line near the obstruction. The distances $m\,d$ $m\,d'$, and also the lines $d\,p'$ $d'\,p$, should be measured, so that $m\,d : d\,p' :: m\,d' : d'\,p$. The lines $m\,p$ $m\,p'$ should be traced and measured to P and P', so that $m\,d : d\,P' :: m\,d' : d'\,P$. The line $P\,P'$ should be measured and divided in the point a', so that

$$P\,a' = \frac{d'\,a \times P\,P'}{d\,d'}.$$

The point a will be a point in the line. From a' any convenient distance $a'\,c'$ may be measured in $a'\,P'$, so that p' may be visible from c'. In the line $a'\,P$ the point c should be determined by measurement, making

$$a'\,c = \frac{a'\,c' \times a\,d'}{a\,d}.$$

The lines $p'\,c'$ $p\,c$ should be traced to intersect in m', which will be also a point in the forward part of the line. The accuracy of the determinations may be verified by the measurement of $c\,m'$ $c'\,m'$, which should be proportionals to $m\,d'$ $m\,d$ or $c'\,m' \times m\,d' = c\,m' \times m\,d$. The line may be traced backwards to the obstruction on the

line $a'm'$, and forward from m' to the end of the line. Lines so long as not to be accurately traceable by the eye should not be selected or adopted in a survey with the chain. On long lines the trace should be made with a theodolite, or other suitable instrument, to obtain or determine points in the line with the degree of accuracy required for the purposes of the survey. Tracing lines with the theodolite and other instruments will be described in a subsequent chapter.

Surveying for Detail.

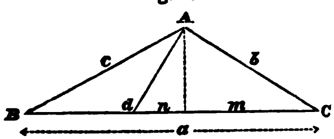
The surveyor, on commencing the survey of a detail triangle, should measure its primary reference and split lines, and make the other necessary measurements for the detail on them. The measured distances should be entered on the field-book at the time of reading the measurements in the manner hereafter described. With a view to render unnecessary the defacing for correction of the original entries on the field-book, the measurements on the secondary reference lines, and the farther survey of detail, should not be taken up until the accuracy of the measurements of the primary reference lines and the split line be sufficiently verified.

The verification of the lengths of the sides and split line of a triangle may be made, with sufficient accuracy, by properly plotting to a large scale the side on which the reference point of the split line may be; and from the extremities and the reference point so plotted, the lengths of the other sides and split line should be elegantly and accurately scribed from their respective points in the plotted line as centres. The traces of the tracing or scribing point of the compasses for the three scribed distances will have a common point of intersection if the measurements be accurately made and the

plotted lengths correctly laid down. In many localities, particularly in outlying districts, this verification cannot be conveniently made. The verification may be otherwise made as follows:—

In the annexed diagram (fig. 4) let the sides of the triangle $A\,B\,C$ be represented by $a\;b\;c$, and let l represent the split line $A\,d$. The lines $a\;b\;c\;l$ and the segments $B\,d\;\;d\,C$ are given by chain measurements.

Fig. 4.



Now let m be the distance from C to a perpendicular let fall from A upon $B\,C$, and n the distance from the perpendicular point in $B\,C$ to d. Then

$$m=\frac{a^2+b^2-c^2}{2a}.\quad \text{And}$$

$$l=(b^2+n^2-m^2)^{\frac{1}{2}}.$$

This gives l in terms of the measured sides by means of the auxiliary distance m. This value of l should agree with its measured length if the length of the latter and the other lines be correctly obtained. If the computed and measured lengths of l be not in close agreement, the split line should be remeasured before seeking the error in the other lines. If the error be not in the split line, the reference point d may be computed from the formula—

$$n=(l^2+m^2-b^2)^{\frac{1}{2}}.$$

And hence $m+n$ will give the measurement of the point d from C. Should the reference point d be found, on remeasurement of $C\,d$, to be in error to this amount, the verification will be sufficiently made. If, however, the error be not found at d or in l, the other lines must be

remeasured, and the error removed before proceeding with the survey of detail.

After the verification of the measurements of the primary reference lines of the triangle, the surveyor should proceed with the secondary reference lines for the survey of detail; these should be selected to coincide as near as may be with the detail to be surveyed on them. When the coincidence is perfect, the measurements for the extreme points of the detail determines its position and magnitude. If there be not perfect coincidence, the deviation from the reference line should be ascertained by the measurement of perpendiculars in the horizontal, having one end in the reference line and the other in the detail. The number and position of these perpendiculars will depend on the form and extent of the detail and the objects of the survey. In surveys for areas and accurate maps the perpendiculars should be so selected as to have the detail between each pair of consecutive perpendiculars a straight line.

The accurate determination, trace, and measurements of perpendiculars to reference lines in the horizontal is of much practical importance, and many expedients have been from time to time devised to facilitate the determination of perpendiculars; but most of these have fallen into disuse by skilful and experienced surveyors. The optical square and crosshead are amongst the most convenient instruments for the purpose; but even these fail to meet the requirements under circumstances of frequent occurrence. The following practice finds most favour with skilful surveyors, but it requires considerable practical training to give the necessary degree of accuracy to the determinations:—

The surveyor, standing over the chain adjusted in the

reference line, with his face towards the detail to be surveyed, should hold transversely over the chain a straight-edge, so that the eye shall be in the vertical plane with it, and the four angles which the straight-edge makes with the chain shall be apparently equal. Then will the prolongation of the line of the straight-edge be the trace of a perpendicular to the chain in the horizontal from the point over which it was held. A straight arrow, or the offset staff, is generally used as the straight-edge for the purpose. If the chain be not in the horizontal at the reference point for the perpendicular, it may be brought to it by supporting a few links of it, at the lower side, with the toe of the boot, without otherwise disturbing it in the line, until the perpendicular be determined and traced. Perpendiculars carefully traced in this manner, when not more than a chain in length, and the ground in the direction of the reference line horizontal, will be found sufficiently accurate for most practical purposes.

Perpendiculars for special purposes, and in some other cases, may exceed a chain in length. In the former, or special cases, the perpendicular is made a reference line, to which are referred other perpendiculars for the survey of detail not otherwise easily obtained. In those cases the direction of the perpendicular is required with great accuracy. The mode of erecting perpendiculars described above, except in skilful and experienced hands, generally fails to give the direction with sufficient accuracy. And in other cases, not of unfrequent occurrence, where the detail is much above or below the level of the reference point of the perpendicular, the method described above cannot be adopted with complete success. The same objection applies with greater force to the use of the optical square and crosshead, but more especially

the former. In such situations the following method will give satisfactory results :—

Two known points in the reference line, about 30 links apart, should be selected, between which it is judged the perpendicular from the feature point in the detail will fall. To one of these points corresponding link-rings near the ends of the chain should be attached with an arrow driven firmly into the ground, or otherwise; to the other the middle of the chain should be attached in like manner. Then, by drawing out the chain on either side, to have corresponding link-rings in the vertices of the similar triangles so formed, so that the prolongation of the trace of the line joining these vertices will pass through the feature point in the detail, the line so traced will be the required perpendicular, and the intersection of the reference line will be its reference point. A perpendicular to the reference line from a given reference point may be accurately traced as above by taking the vertices of the similar triangles, so that a line joining them will pass through the given reference point. A perpendicular to a reference line, from a given point in the same, may be more conveniently erected as follows :—Equal distances should be measured in the reference line from the given point, so that the given point may be the bisection of the whole measured distance. To the extreme points the rings of corresponding links near the ends of the chain should be attached, as above directed, and the chain drawn out by the 50-mark, or middle point. A line traced through the vertex of the isosceles triangle so formed and the given reference point will be the required perpendicular. A perpendicular may be also raised from a given point in a straight line by the construction of a right-angled triangle, one leg of which, terminating in the given point, shall be in the reference

line, and the other leg in the perpendicular. The sides of the triangle must be multiples of 3, 4, and 5.

These modes for determining and erecting perpendiculars are sufficiently accurate for most practical purposes, if the points in the reference line, to which the corresponding links near the ends of the chain are attached, be on the same level.

It may be seen that a perpendicular traced as above described is in a plane, to which the trace of the reference line between the selected points shall be a normal. Hence, when the latter is horizontal, the plane of the perpendicular will be vertical, and its trace on the surface will be that of the true perpendicular for the purpose of the survey. To the general principle just stated there is one exception:—If the trace of the reference line between the selected points be not horizontal; and if the trace on the surface of the plane of the perpendicular be horizontal, this trace will be that of the true perpendicular. On sidelong ground perpendiculars in the horizontal cannot be determined or erected with the required degree of accuracy by any of the methods with the chain described above. Under these circumstances, and generally:—

If the reference line be more than 100 links from the detail, for any considerable distance, an offset triangle should be laid down, on the sides of which the detail may be surveyed on perpendiculars of moderate length. These perpendiculars may be determined and traced in the manner first described. Except in those cases where the reference line is of considerable length, it will be advisable to select a second reference line, instead of the offset triangle, unless the form of the detail to be surveyed specially favours the triangle.

In general survey operations it is desirable that the

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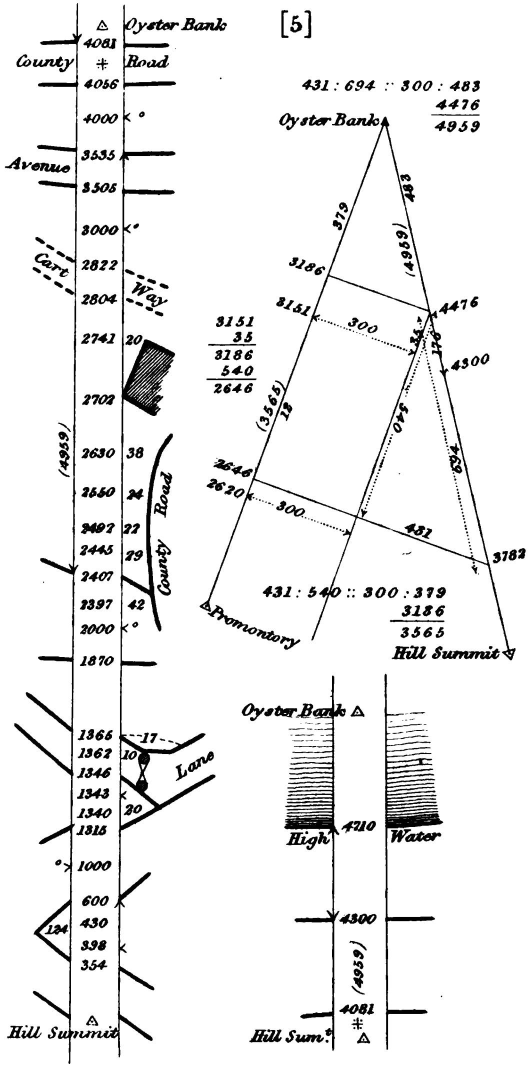

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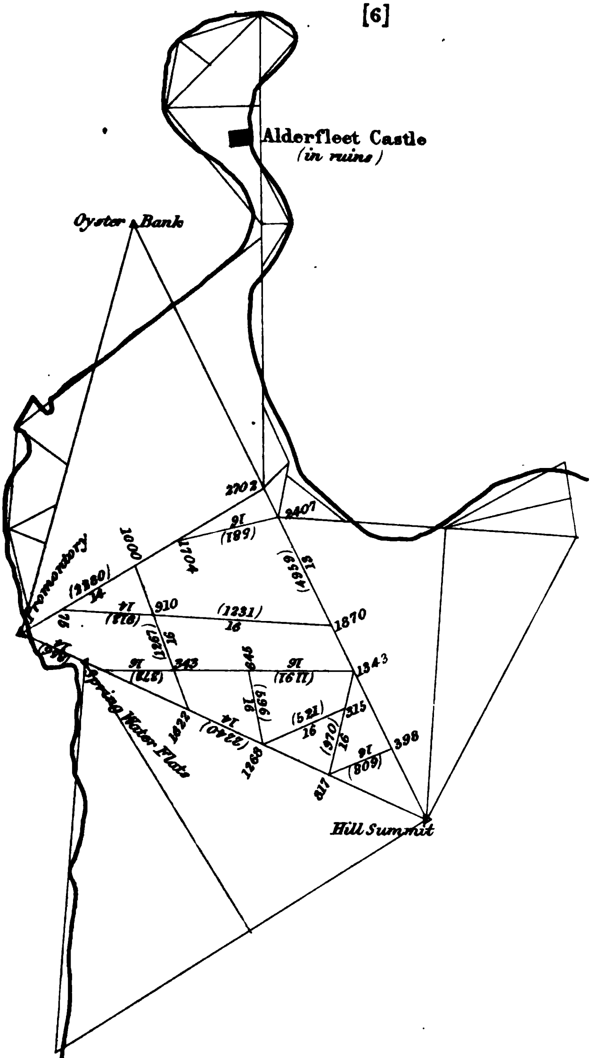
chain be not disturbed from its position in the reference line, and hence all perpendiculars and other short distances should be measured with the offset staff or tape line. When the detail is surrounded by thorny shrubs, or otherwise difficult of access, as in the case of trenches filled with water, &c., the use of the offset staff will be attended with advantage, as it enables the surveyor to complete the measurements to the detail with accuracy and expedition, and without inconvenience, injury, or danger. This may be done by forcing the end of the staff through the thorny shrubs to the detail, or by placing it on the middle or farther edge of drains, &c. The difficulties and inconveniences attending the measurement of perpendiculars with the chain or tape line in those cases induce the very objectionable practice of guessing part of the distance. The surveyor should not place on his field-book any measurement not properly and accurately made.

The field-book may be in size about ten by eight inches when closed. It should be made of good plain paper, interleaved with blotting-paper. The entries should be made in ink, and erasures should be strictly avoided. In the correction of a measurement, any error in booking should be struck out with the pen in a manner not to render it illegible. The correct entry should be made above the one struck out—thus: $37\overset{25}{\cancel{75}}$. The entries for intermediate measurements on reference lines or perpendiculars should be made across the direction of the line to their respective points. The length of the whole line or perpendicular should be entered along the line of direction. The measurements of the primary and long secondary reference lines should be entered on the field-book in the manner generally adopted for independent

lines. The entry of the measurements should be commenced at the bottom of the line or column, to which column the pickets should be entered as marked on the ground, to the right, left, forward, or backward, as the case may be. An inspection of the annexed copy of field-book for primary reference lines (fig. 5) will suffice in this place.

The field-book should indicate the order and manner in which the measurements are made, and be, in fact, a complete sketch-plan, with measurements, of the district surveyed. Each page of the field-book should represent a section or part of the district, and show the original entries on it between two faint parallel lines, as shown in the illustrations (figs. 5, 6, and 7). The entries copied from another page for the purpose of making connections and other reference should be shown on a single faint line. The lengths of the reference lines, and the pages on which the original entries are made, should be written, for the purpose of reference, on the copied line.

The entries for the measurements on a reference line, whether for reference or detail, should be placed on the field-book, as already stated, between the faint parallel lines. The entries for the lengths of perpendiculars from the reference line to the detail should be entered to their respective reference points, right or left, as the case may be. The space between the faint parallel lines must be regarded as of no absolute width. The measurements for the detail, except where distinctly shown otherwise, should be made on perpendiculars to the reference line. The unbroken deep lines on the field-book illustrations referred to shows the defined detail. The undefined detail should be shown by dotted lines.



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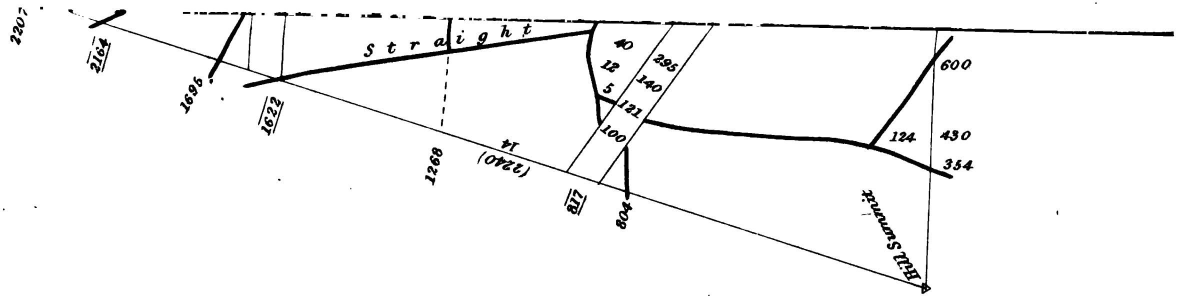
2207
2164
1696
1622
Straight
1268
(2240)
14
817
804
40
12
5
295
140
121
100
600
124
430
354
Hill Summit

Besides the sketches and measurements entered on the field-book, the date of the survey and names of the survey party should be placed on each page, and also the date when any subsequent correction may have been made, and the name of the party by whom it was made.

Each field-book should contain sketch diagrams of all the reference lines of the survey entered on it. The diagram of lines should show the measurements of the lengths of the reference lines and their reference points. The illustration (fig. 8) is a sketch diagram of reference lines, with only that part corresponding to the illustrations fig. 6 or fig. 7 complete.

In surveys for a map only, the position of the extreme points of straight detail is all that is required to lay down on paper a correct representation of it. The position of these points may be determined by independent measurements on, or referred to, the same or different reference lines. From this it may be seen that a subdivision of the primary triangle by other triangles and quadrilaterals is admissible for the purpose. An inspection of the illustration annexed (fig. 6) will enable the reader to understand the manner in which the selection of quadrilaterals in connection with triangles may be adopted for the purpose of the survey. This is technically termed 'blocking.'

When it is required to find the area of the several parcels of a district by computation from the survey notes or field-book, a more perfect survey must be made, in which 'blocking' is inadmissible. In surveys for this purpose the measurements should be made with great accuracy, and so selected as to give sufficient data for the purpose. The geometric lines should give the data in detail for com-

puting the areas of the figures bounded by these lines. As these figures will seldom, if ever, coincide with the detail, it becomes necessary to find the areas bounded by the detail and the geometric lines. This must be found from the lengths of the perpendiculars surveying the detail and their reference points in the reference or geometric line. The separate quadrilateral elements of the area between the reference line and the detail will be bounded by the parallel perpendiculars on opposite sides, and the detail and reference line on the remaining sides. As it will rarely happen that the reference points of, or in, a reference line will be in the detail surveyed on it, there arises a necessity to connect the detail measurements at the last or first point surveyed on adjacent reference lines, as the case may be. An inspection of the annexed (condensed) page of field-book (fig. 7) will show the connection of the detail measurements at the ultimate point in the detail, and the manner of booking surveys for this purpose.

It may be seen that the lengths of the reference lines are placed in brackets, and that the entries for reference points are placed between parallel lines. These important measurements are so placed to distinguish them, not only for the purposes of computation, but also for facilitating the reference in the field and office.

The example field-books for a map and for areas, and the skeleton diagram of lines, are given here in connection with the foregoing, not because of the difficulties attending the making of the survey, which principally occurred in the parts not illustrated, but because the author has already acceded to the request of the professors of Civil Engineering in the Queen's Colleges in Ireland to allow a copy of the field-book of the survey to be lithographed for the use of the engineering students of these colleges.

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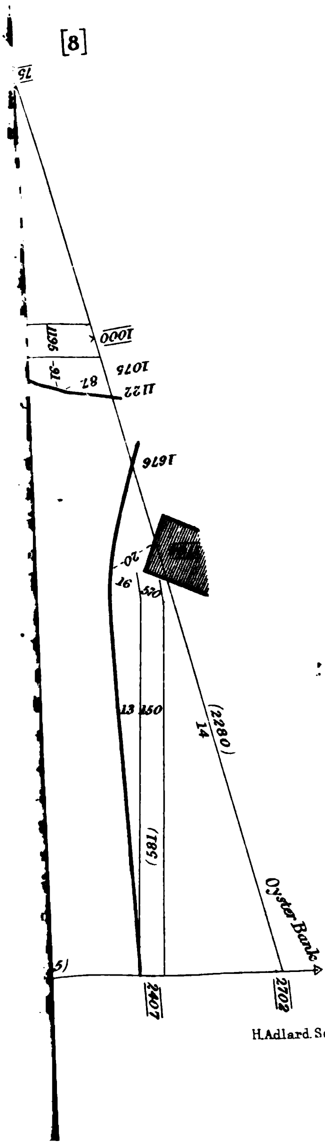

75
1000
1195
1075
91
87
1122
1676
20
16
520
13
150
(2280)
14
(185)
Oyster Bank
51
2407
2702

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Obstructions to the Making of Direct Measurements. —Inaccessible Distances.

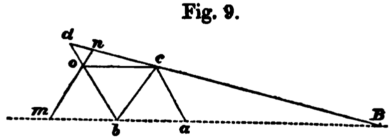
In the preceding pages on the measurement of lines and distances, it was tacitly assumed that the lines to be measured were accessible throughout. In practice this is very frequently not the case, particularly on long reference lines for the survey of districts of considerable extent. Thickets, buildings, rivers, lakes, or other directly impassable obstructions, will sometimes be on a reference line, and so interfere with the direct continuous measurement of the line. The surveyor should be prepared to find the length in the line occupied by such obstructions with sufficient accuracy, that the whole length of the reference line may be correctly obtained by the survey. It would be impossible, even if it were desirable, to anticipate the many circumstances which occur in connection with inaccessible distances; it is therefore proposed to consider the best modes for finding these distances with considerable accuracy, and which are of such character as to be applicable in practice under various combinations of circumstances.

The surface configuration of the ground, the form and character of the surface obstructions in each case, must be considered by the surveyor in the field, who will have to select the mode which shall be best suited to the circumstances. It must not, however, be lost sight of that constructions for the purpose of finding inaccessible distances are well or ill conditioned according as slight errors in the laying down of the construction, or measured diagram, may effect the results to an insignificant or great amount. Constructions or diagrams which are small in their measured parts as compared with the inaccessible distance are essentially of the class *ill con-*

conditioned; and constructions or diagrams which have the points determining the direction or position of a line, themselves determined by the intersection of other lines, meeting at a small acute angle, are necessarily *ill conditioned*. So also are those constructions *ill conditioned* in which the proportional lines for computation make small acute angles with one another.* From this it may be seen that *well conditioned* constructions should have the determining lines equal, or nearly equal, to the inaccessible distance, and the angles made by the intersecting and determining lines not very acute, but as nearly equal to right angles as may be. In every case it is desirable that the angle should not be less than 30° to 40°. When practicable, inaccessible distances should be found by the measurement of an equal or proportional accessible distance, in preference to finding the distance by computation from a measured diagram. In either case the measured or computed distance should be verified before the distance can be considered as properly found.

* These remarks may be illustrated by reference to a method proposed by Mr. Baker, in his treatise (published by Mr. Weale), page 35. The principle is correct, but the application of it is very faulty. In the figure (9) the triangles, $d\,o\,c$, $c\,a\,B$, are similar.

Fig. 9.



The distance $a\,B$ is found by the proportion $d\,o : o\,c :: c\,d : a\,B$. It would be preferable to find $b\,B$ by the proportion $n\,o : o\,c :: c\,b : b\,B$. Mr. Baker makes $o\,c = c\,a = c\,b = 50$ links; so that in practice $o\,c$ must be very small as compared with $a\,B$, and the determining angles at B and d very acute. These two very objectionable features are combined in this construction as applied by Mr. Baker.

There are many modes put forward for finding inaccessible distances which shall be omitted, as of little or no service to the surveyor in the field. In the following modes for finding inaccessible distances by measurement or computation, all the measurements should be made to the horizontal, except where stated otherwise.

In the measurement of a reference line, the point for making the chain length will not unfrequently be inaccessible, as when that point comes to be in dense thorny shrubs, the middle of a deep drain, &c. In such cases the leader should take up the chain at the last convenient accessible point in the line before coming to the obstruction, and mark it with an arrow. The follower should take the measurement of this point. Then the leader should pass his end of the chain to the forward side of the obstruction, which should be less than a chain length on the line.* After laying down a forward chain length from the last measured point, the leader should draw forward the chain in the line, and lay down from the arrow marking the chain length the complement of the broken chain length. The arrow marking the chain length should be removed to the forward end of the complement laid down. This arrow will then mark correctly a chain length in the measurement of the line. [If the measured distance to the point on the

* Passing one end of the chain over fences, thickets, rivers, &c., less than one chain across, may be conveniently done by making it up from one end; and when so made up, throwing it in the proper direction, holding at the same time one of the handles in the left hand. The chain may be passed through thorn hedgerows, &c., by slipping the handle of the chain on to the handle of the spade and pushing the latter through at the proper place, so that it may be drawn out at the other side of the hedgerow, &c.

broken chain length at the obstruction be 2580, the measurements in the line would stand thus:—

$$2580+100+20=2700,$$

the measurement of the point marked with the arrow at the forward end of the complement laid down.]

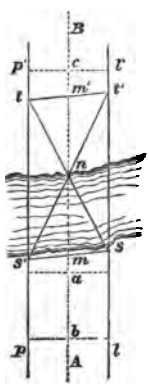
Inaccessible Distances.—Wide Rivers.

When the measurement of a reference line is obstructed by obstacles such as wide rivers, the inaccessible part may be found so that the entries on the field-book for use in the office department of the survey shall be continuous.

The following method will be found to apply over a wide range of circumstances:—

Let $A B$ (fig. 10) be the reference line, and $m n$ the inaccessible distance. The surveyor proceeding from A in the direction of B should complete the measurements to m, the farthest accessible point on the A side of the river. At two points, $a b$, sufficiently far apart for tracing a line accurately, perpendiculars should be erected, and traced both ways. From a and b equal perpendiculars, not less than the half of $m n$, should be measured. Through the extremities of the perpendiculars, equidistant parallel lines, $l l'$, $p p'$, should be traced. At s and s' the points of intersection of the parallel lines $l l'$, $p p'$, and the line $s m s'$, ranging rods should be set up vertically, and the equal lines $s m$, $m s'$ measured for the verification of the

Fig. 10.

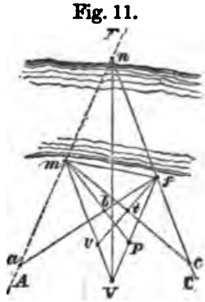


construction on the A side of the river. A ranging rod should be set up vertically at n in the reference line, and at c other perpendiculars, equal to those laid down at $a\ b$, should be laid down to verify the trace of the parallel lines $l\ l'$, $p\ p'$, on the B side of the river. The lines $s'\ n$, $s\ n$ should be traced to intersect the parallel lines at t' and t. The line $t\,t'$ should be measured and found equal to $s\ s'$, and the part $t\ m'$ should be equal to $m'\ t'$ if the lines and distances be accurately traced and measured. Then will $m'\ n$ be equal to $m\ n$. The surveyor should hold to m' the link on the chain which measured the point m, and continue the measurements from m' to n, without measuring for detail between these points. The measurement for the point n so made will be the same as if the measurements could have been continued from m to n.

Inaccessible distances of considerable extent may be found by the measurement of equal distances determined as follows:—

Let $m\ n$ (fig. 11) be the inaccessible distance in the reference line $A\ B$. From a, a point in $A\ B$, a straight line, $a\ b\ f$, should be measured, and from m another point in the same line, $m\ b\ p$, should be measured, making $b\ p = \frac{m\ b \times b\ f}{a\ b}$. Then will the line $f\,p$ be parallel to $A\ B$. Now from c, a point in the line $n\ f$ produced, the line $c\ t\ m$ should be measured, and from f the line $f\ t\ v$ should be measured, making $t\ v = \frac{m\ t \times t\ f}{c\ t.}$.

Fig. 11.



Then will $m\ v$ be parallel to $n\ c$. The lines $f\,p$ and $m\ v$ should be produced to meet in V. In the parallelogram

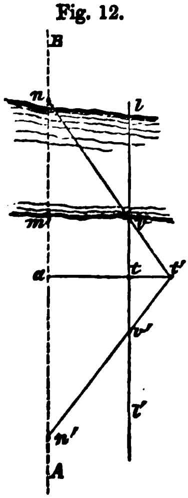
$m n f V$ so constructed, the accessible distance $f V$ is equal to the inaccessible distance $m n$, and $m V = n f$.

If $C n$ be a reference line, the point n and the inaccessible distances, $m n$, $f n$; in the reference lines $A n$, $C n$, may be determined by the equal and accessible lines of the construction. The line $f m$ should be measured and bisected. Then will the point of bisection be found to be in the line $V n$ if the construction be correctly laid down. This test will verify the correct determination of the inaccessible distance if the accessible lines be accurately measured.

By another construction, requiring fewer measurements than fig. 11, but affording no means of verification, an accessible distance equal to the inaccessible may be determined as follows:—

At a convenient distance from the reference line, $A B$ (fig. 12), a line $l l'$ should be traced, parallel to $A B$, by one of the foregoing methods. From t, a point in $l l'$, equal distances $t v$, $t v'$, should be measured, and at the points $v' v$ ranging rods should be set up vertically. The line $n v$ should be produced to any convenient point t'. Then, by producing the lines $t' v'$, $t' t$, to intersect $A B$ in n' and a, the accessible distance $a n'$ will be equal to $a n$, which includes the inaccessible distance $m n$. The angle at n should not be less than 30°. If $t v'$ be made a sub-multiple or any definite part of $t v$, $a n'$ will be a like sub-multiple, or proportional part, of $a n$.

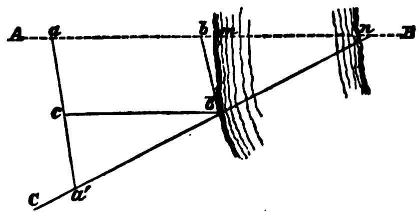
Fig. 12.



By (arithmetical) computation of a side in similar tri-

angles we are enabled to find inaccessible distances in the field without serious loss of time or interruption to the progress of the survey. In the illustration (fig. 5) or field-book, showing the survey on a primary reference line, is given an example of the practical application of the following method for finding inaccessible distances:

Fig. 13.



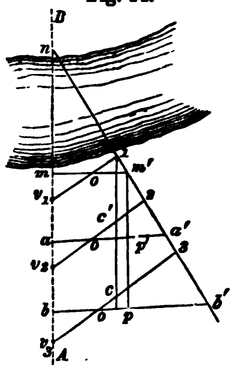
Let $A\,B$ (fig. 13) be the reference line, $m\,n$ the inaccessible distance. At a and b, in $A\,B$, on level or equally inclined ground, perpendiculars should be erected and traced to $a'\,b'$, points in the line $C\,n$, and $a'\,c$ laid down equal to $a\,a' - b\,b'$. Then will $c\,b'$ be equal to $a\,b$, and from the similar triangles $c\,a'\,b'$, $b\,b'\,n$, we find $b\,n = \frac{c\,b' \times b\,b'}{c\,a'}$ and $n\,b' = \frac{a'\,b' \times b\,b'}{c\,a'}$. These computed distances include the inaccessible parts of the lines A B, $C\,n$. The distances computed from these formula are not checked by anything in the construction.

In some cases the following method will be found preferable to that just given above.

At $m\ a\ b$, points in the reference line $A\,B$ (fig. 14), parallel lines or perpendiculars to $A\,B$ should be ranged by one of the methods already described. For the more accurate determination of the position of the parallel lines, it is desirable that the points be so selected that

the ground shall be level, or equally inclined, in the direction of the reference line. Under these conditions the parallel lines may, perhaps, be more conveniently laid down as follow: — The equal distances $m\,v_1$, $a\,v_2$, $b\,v_3$ (= 30) should be laid down in $A\,B$ on the same side of $m\,a\,b$. To the points $m\,v_1$, $a\,v_2$, &c., corresponding points near the end of the chain should be attached with arrows, as already described, and the vertices $o\ o$, &c., determined by drawing out the chain by the same link-ring. The line $m\,o$ should be traced to m', and ranging rods set up vertically at n and m'. The lines $a\,o$, $b\,o$, $v_1\,o$, $v_2 o$, $v_3 o$, should be traced to intersect the line $n\,m'$ in the points $a'\,b'$, 1, 2, and 3. By laying down $v_3\,c$ in $v_3\,1$ equal to $v_1\,1$, the line $c\,1$ will be equal and parallel to $v_1\,v_3$. In like manner the line $p\,b$ should be laid down equal to $m\,m'$, then $p\,m' = c\,1 = m\,b$. This will be a sufficient check on the correct laying down of the construction or diagram. Now we have from similar triangles

Fig. 14.



$$m\,n = \frac{b\,m \times m\,m'}{p\,b'};\ \text{or}\ m\,n = \frac{a\,m \times m\,m'}{p'\,a'}.$$

In many cases it will be preferable to take the alternative diagram. Then $v_1 n = \dfrac{v_1\,1 \times c\,1}{c\,3}$ and $v_1\,n = \dfrac{v_1 \times c'\,1}{c'\,2}$.

These results verify the determination of the distances by computation. Another check on the accuracy of the results may be applied by omitting the parallel lines $a\,a'$, $v_2\,2$, and taking both constructions thus:

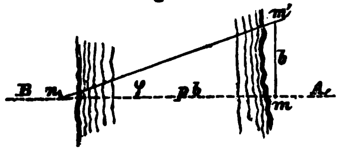
$$m\,n\,(= \frac{b\,m \times m\,m'}{p\,b'}) = \frac{v\,1 \times c\,1}{c\,3} - m\,v_1.$$

In applying this construction the angle $m\,n\,m'$ should not be very acute. If the inaccessible distance be considerable, or the grounds at $m\,a\,b$ irregular, it will be decidedly preferable to lay down the construction, or diagram, as follows:—A line c 1 should be traced parallel to $A\,B$, by one of the methods already described, so that the distance between the parallel lines may not be less than half $m\,n$. From 1, a point in the parallel line, the distances 1 c', 1 c should be laid down equal to $m\,a$, $m\,b$ respectively. Again, the line n 1 should be traced to any convenient point m', and the line $m'p$ traced parallel to 1 c. The distances $m'\,p'$, $m'\,p$ should be laid down from the point m', in the line $m'\,p$, equal to $m\,a$, $m\,b$ respectively. From the points $m\,a\,b$ any convenient equal distances $m\,v_1$, $a\,v_2$, $b\,v_3$ should be laid down in $A\,B$ on the same side of these points. Now the lines $v_3\,c_3$, $v_2\,c'_2$, v_1 1, will be parallel, as will also be the lines traced through $m\,m'$, $a\,p'$, $b\,p$. The parallel lines so determined should be traced to intersect the trace of the line n 1, produced in the points shown on the diagram. The distances in these parallel lines between the lines parallel to $A\,B$ and the line $b'\,n$ should be measured. Then the computation will be as in the formula.

INACCESSIBLE DISTANCES, *with the aid of the theodolite.*

If the angle at n (fig. 15) be laid off so that $m\,n$ shall be a multiple of $m\,m'$, the angle at m being a right angle, the inaccessible distance may be found by an easy computation, if $m\,m'$ be accurately measured. Let the angle at n be represented by ϕ, and the perpendicular $m\,m'$ by b, then the distance $m\,n = b$ cot ϕ—a general formula—and

Fig. 15.

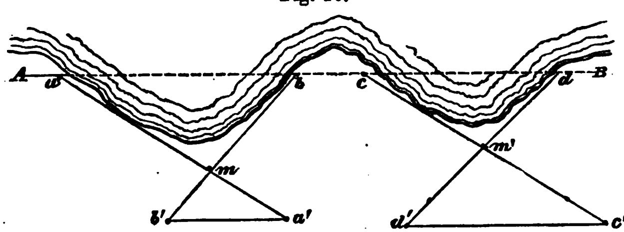


$\cot\phi\,(=\frac{m\,n}{b}) = p$, the multiplier of the perpendicular. If p be taken a whole number, 1, 2, 3, &c., the angle ϕ may be computed from the formula for these values of p. If the angle ϕ be laid off for $p = 3$ then $b \times 3 = m\,n$. This is a very convenient mode of computing the distance: the angle ϕ and the right angle at m should be laid off with a theodolite, by which the lines should be also traced to determine the point m'.

There is another class of inaccessible distances, the extreme points of which are accessible from a third point; these frequently occur on reference lines for the survey of water margins, rivers, thickets, &c. The following explanations and directions will suffice for most cases of this class which are likely to occur in open country.

Let $A\,B$ (fig. 16) be a reference line on which $a\,b$, $c\,d$ are inaccessible distances, the extreme points of which are accessible from the points $m\,m'$ respectively.

Fig. 16.



Through the accessible points $m\,m'$ straight lines from $a\,b$, $c\,d$ should be traced and measured to the points $a'\,b$ $c'\,d'$, so that $a\,m : b\,m :: a'\,m : b'\,m$ and $c\,m' : d\,m' :: c'\,m' : d'\,m'$. Also $a\,m : m\,a' :: a\,b : a'\,b'$. From this it may be seen that whatever multiple $a\,m$ is of $m\,a'$ $a'\,b$

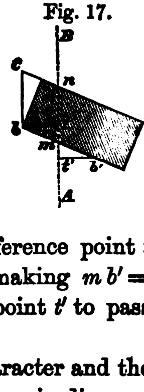
will be the same multiple of $a' b'$. The line $a' b'$ should be measured, and the corresponding distance $a b$ computed from the formula $a b = \frac{a m \times a' b'}{m a'}$. In general practice $m a'$ is made equal to $m a$, so that $a' b' = a b$. If this be found inconvenient, $m' c'$ may be made $= \frac{c m'}{2}$, so that $d' c' \times 2 = c d$.

The inaccessible distances in buildings and other obstructions raised above the ground may be found by this method.

When the obstruction is a single rectangular building the inaccessible distance may be more conveniently found as follows:—

Let $A B$ (fig. 17) be the reference line and $m n$ the inaccessible distance. The surveyor, proceeding with the measurements from A to B, should take up the measure of m. He should measure $m b$ and lay down from n the equal distance $n c$, so that $b c$ shall be equal and parallel to $m n$. The point on the chain measuring m should be held to b and the measurements continued to c. The ascertained measurement for c will be the measurement for n. The inaccessible reference point t and perpendicular $t b$ may be found by making $m b' = m b$ and raising a perpendicular from the point t' to pass through b'. Then $t' b' = t b$ and $m t' = m t$.

Fig. 17.



If the obstruction be of an irregular character and the trace of the reference line be made as shown in diagram (fig. 3), the inaccessible distance $a a'$ may be computed from the measurements for tracing the line. In this case

$$a a' \left(= \frac{m a \times P d'}{m d'} \right) = \frac{m a \times P' d}{m d}.$$

In the case of reference lines for the survey of detail, parts of which may be inaccessible, adjacent to accessible grounds, such as not unfrequently occurs on lines for the survey of rivers, water margins, thickets, &c., a parallel line should be traced on the accessible ground on which the survey of the detail may be made. The parallel distance line should be clearly shown and fully referenced to the reference line. This may be done by erecting a perpendicular, from a given point, to the reference line and tracing it to intersect the parallel line in the corresponding point. From the latter point the measurements should be continued in the parallel reference line for the survey of detail until the inaccessible part of the reference line be passed. A measured point in the parallel reference line should be selected as the reference point of a perpendicular, which should be traced to intersect the reference line in a corresponding reference point. The entry of the measurement for the reference point in one line should be also entered to the corresponding reference point in the other line, from which the measurements should be continued in the latter, as described above.

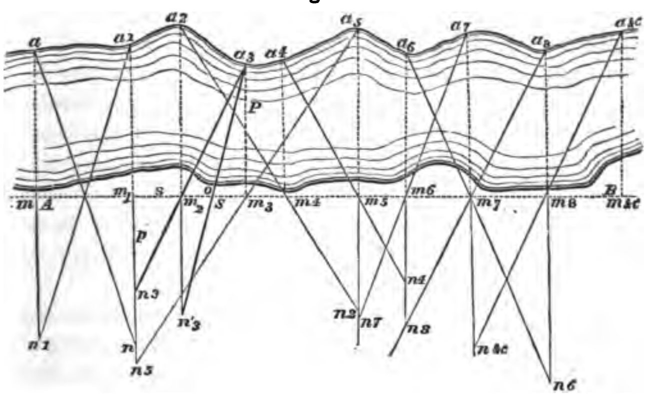
In other cases, such as for wide rivers, the centre of which may be part of the limits of a district, and for other purposes, the survey of the distant inaccessible side may be required. And also when the water line of a marshy lake, or like inaccessible detail, is to be surveyed, the service may be carried out as follows:—

Let AB (fig. 18) be the reference line for the measurement of the inaccessible detail. And let a, b, a_2, a_3, &c. be the feature points. It is required to find the position of these points, so that they may be correctly laid down on the map.

The ground along AB is assumed to be open, and free

from surface encumbrance. From a, a perpendicular $a\,m$, should be let fall on $A\,B$, or, in other words, the reference point (m) of the perpendicular from a should be determined in $A\,B$, as already explained, and the perpendicular traced on the other side of $A\,B$. The reference

Fig. 18.



points m_1, m_2, &c., in $A\,B$ for perpendiculars to the other inaccessible feature points a_1, a_2, a_3, &c., should be determined in like manner, and the perpendiculars traced as may be required. Now by tracing the line $a_3\,m_2$ to intersect $a_1\,m_1$, produced in n_3, we have $m_1\,n_3 : a_3\,m_3 :: m_1\,m_2 : m_2\,m_3$. And hence

$$\frac{m_2\,m_3 \times m_1\,n_3}{m_1\,m_2} = a_3\,m_3,$$

the perpendicular for a_3. Putting $P\,p\;S\,s$ for the distances we have the general formula

$$\frac{p \times S}{s} = P.$$

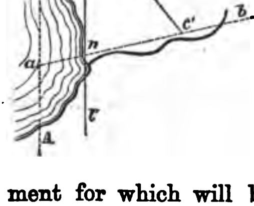
If $m_2\,m_3$ be bisected in o, and the line $a_3\,o$ be traced to intersect $a_2\,m_2$, produced in n'_3, then, from similar and

equal triangles, $m_2 n'_3$ ($=a_3 m_3$) the inaccessible distance. An inspection of the diagram will suffice to show the manner of working out the determination of the lengths of the several perpendiculars. The selection of the particular lines for measurement in these determinations must be left to the surveyor in the field. In this selection due regard must, however, be had to the conditions, already stated, for well conditioned diagrams.

In certain localities or parts of a district it may be not unfrequently advisable to select secondary reference lines, having one or both reference points in the inaccessible parts of other reference lines. In those cases it is necessary to find the measurement for the reference point and the inaccessible distances terminating in it. These measurements may be conveniently found as follows:—

Let $A B$ $a b$ (fig. 19) be the reference lines and a the inaccessible reference point in $A B$. The lines $l l'$ $m n'$ should be traced parallel to $A B$ $a b$ respectively. These parallel lines should be traced to intersect $A B$ $a b$ in the points m n respectively, and one another in the point n'. It will be seen that $a m n' n$ is a parallelogram. To find the measurement for the point a in $a b$, the point on the chain giving the measurement for the point n should be applied to n' and the survey continued to m, the measurement for which will be that for $a - (a n = m n')$. In like manner if the point on the chain for the measurement of m be applied to n' the measurements may be

Fig. 19.



continued to n. As $m\,a = n'\,n$, the measurement for n so made will be the measurement for a in $B\,A$.

For the verification of the construction and measurements, let a line $c\,n'\,c'$ be traced, and let the lines $c\,n'\,c'$, $c'\,n$, and $c\,m$ be measured. Then, from similar triangles,

$$\frac{c'c \times n\,n'}{c'\,n'} = c\,a \text{ and } \frac{c\,c' \times m\,n'}{c\,n'} - n\,c' = a\,n.$$

The Reference of Reference Points.

The measurement of the reference point of a reference line, if the point be not surveyed previous to the measurement of the reference line terminating in it, should be found by measurements from it to well defined and previously surveyed pickets, or points, on either side, in the line in which it is an intermediate point, so that the measurement of the reference point may be found from the measurements of two independent points. If the difference between the greater measurement and the reference point distance to its picket, or point, be equal to the sum of the other reference point distance and the lesser measurement, the measurement of the reference point will be correctly found. It should be also found that the difference between the measurements for the pickets on either side of the reference point is equal to the sum of the reference point distances. A few figures will make this clear. Let the reference point distances back and forward be 140 and 204 respectively, and let the measurement for the backward picket be 5,937, and for the forward picket 6,281. Then 6,281 − 204 = 6,077, the measurement of the reference point as found from the forward picket, and 5,937 + 140 = 6,077, the same measurement determined from the backward picket. Also 6,281 − 5,937 = 140 + 204. This verifies

the accuracy of the arithmetical reduction and the measurement of the reference point.

For the purpose of readily distinguishing the reference points of the principal secondary reference lines, double picket marks should be made on the ground, and shown on the field-book to these points.

The boundaries of important primary or secondary districts, such as counties, baronies, parishes, &c., should be shown in character on the field-book, together with an initial description of the boundary detail. In this description the following contractions may be used:—*w* for wall, *f* fence, *s* stream, *h* hedgerow, *r* river, *d* drain, N, S, E, W, north, south, east, west; also T top, F face, C centre, B bank. Combining these, a boundary may be described for some particular case, thus:—W B r, for west bank of river, and N F w, north face of wall.

CHAPTER III.

SURVEYING WITH THE CHAIN AND AN ANGULAR INSTRUMENT —GEOMETRICAL FIGURE.

The explanations and directions given in Chap. II. on surveying with the chain, apply also to the part of the survey operations with the chain which come under consideration in this chapter.

When the district for survey exceeds in extent the limits manageable in a good chain survey, and when it is desirable to obtain a check by computation on the measurements of the primary lines, it is necessary to combine with the chain instruments for measuring angles, furnished with suitable optical contrivances for this important purpose. Instruments, such as theodolites, furnished with telescopes of high power and great range, and which are besides suitable for measuring angles with a high degree of accuracy, enable the surveyor to expand the primary triangulation to an extent depending on the size of the theodolite and the surface configuration of the district. Some angular instruments, such as prismatic compasses, box sextants, &c., are not furnished with telescopes, or otherwise adapted to measure horizontal angles with sufficient accuracy to meet the requirements for the survey of extensive districts with the least practicable error and number of sides in the geometrical figure for the purpose. From this it may be seen that surveys made with the aid of these instruments

are inferior, in point of accuracy, to surveys made with the chain only, when the district admits of a suitable trigonometrical construction for the purpose, and that they are unsuitable for the accurate survey of an extensive district. For reconnoitring purposes, however, and where a high degree of accuracy is not indispensable, the use of either or both of these instruments greatly expedites the survey operations.

The whole of the district to be surveyed should be, if practicable, embraced in the external polygon of the grand or primary lines of the survey, as already stated.

If the sides and angles of the polygon be measured with the chain and an angular instrument, a plot of the lines and a plan of the details surveyed on them may be laid down on paper to a diminished scale, and the enclosed area may be found by computation. The survey so made is denominated a 'traverse.' It may be seen that in this survey there is no certain means of detecting, localising, or correcting errors fallen into in the (field) survey operations. Hence this mode of making surveys is not recommended for the accurate survey of an extensive district.

The verification of the measurement of every line and angle should be made by computation from data obtained through other lines and angles. This verification is practicable if in the polygon another polygon be inscribed, having a lesser number of sides, and its angular points in alternate angular points of the outer polygon, so that a side of the inscribed polygon may be the side of a triangle, whose other sides shall be lines of the outer polygon. If, in the internal polygon so formed, a polygon be inscribed in like manner, thus diminishing the number of sides of the internal polygon, and if the diminution of the sides of the inscribed polygon be so

continued, the ultimate inscribed figure will be a triangle, and all the other resulting figures will be triangles. The sides and angles of the triangles will be adjacent, and their measurement subject to correction by computation. It is obvious from the manner of proceeding that the triangulation so formed would be determined by the form of the external polygon, independently of the surface configuration of the district. Such a triangulation would, for districts of considerable extent and irregularity of surface, be found to present insuperable obstacles to the easy and accurate measurement of the angles and lines, besides the liability to ill-conditioned triangles, particularly in the outer parts. To meet the actual circumstances, in each case, in the best practicable manner, the triangulation should be selected, not only with respect to the form of the district, but also with respect to the surface configuration, in order that the angles may be conveniently measured, and the reference lines traced with accuracy where required. Subject to these conditions, the grand primary triangles should be as few and as nearly equiangular as may be. The points of the district which may not be embraced in the grand primary triangulation may be taken up on secondary offset triangles, thrown out from the grand primary triangles.

The angular, or trigonometrical (trig.), points should be finely marked on permanent objects, and over the trig. points suitable objects for distant observation should be erected. Elevated cones and pyramids are suitable forms for the objects on the trig. points of the grand and primary triangulation of an extensive district. The trig. points may be rendered even more conspicuous at great distances by the use of reflectors, that the sun's, or other powerful light, may be thrown into the telescope

of a distant observer. The trig. points of the secondary triangulation may be sufficiently identified by erecting stout and long poles, near the top of which cross-boards, or other contrivances of light material, should be fixed, to render the trig. point object conspicuous in the locality.

The 'grand primary triangulation' should be divided into a lesser or secondary triangulation, which shall have the trig. points of the former trig. points in the latter. This lesser triangulation, if not sufficiently reduced in size for chain measurements, may be called the 'primary triangulation,' which should be treated in the same manner as described above for the grand primary triangulation. The series of triangles of the third class may be denominated the 'secondary triangulation.' If, from the extent of the district and the difficulties to making accurate measurements with the chain for long lines, the secondary triangles be not the last of the series of subdivisions, the district may be poled for the 'detail triangulation.' In these series of triangulations it is not necessary that the lines of a greater should coincide with those of a lesser triangulation.

In all the triangulations, from the 'grand primary' down to the 'detail,' due regard should be had to essential requirements of well-conditioned triangles and the surface configuration. The selection of the trig. points, particularly in the triangulation, in which these points are numerous, is a very important part of the preliminary operations. The selection should be made so that each trig. point may command a view of the greatest practicable number of surrounding trig. point objects, and that the least angle of a triangle at the trig. point may not be less than about 28°, whilst it is preferable

that the angles shall be as near as may be 60°, as already stated. From this it will be seen that the trig. points will be on leading points or parts in the surface configuration, such as hill tops, and hollows surrounded by elevated ground. In the selection of the trig. points for the detail-survey, another point or element enters into the case, namely, the practicability and facilities for making accurate chain measurements of the sides of the triangles. The selection of the trig. points and the erection of the trig. point objects for the secondary and detail triangulations is technically termed 'poling the district.' The duty of the poling party extends beyond the mere selection of the trig. point. It includes, besides, the proper marking and poling of the point, and such surveys as may be necessary to enable a surveyor to refind it afterwards if necessary.

The permanent mark may be a hole about one inch in diameter jumped into a field stone, which should be securely placed, hole upwards, a few feet below the surface of the ground. The trig. point object should be placed vertically over the trig. point mark. If the object be a pole or a small tree stem, it should be placed on the trig. point, and firmly kept there by a pile of earth or other materials. The position of each trig. point should be determined by measurements to surrounding detail, which should be explained by sketches of the immediate district. These sketches and measurements will enable a party to restore the pole to its first position at any subsequent time should it be accidentally or otherwise removed before the survey operations are complete. In addition to the local measurements, angles should be measured, with the prismatic compass or box-sextant, to other trig. points already poled, so that a diagram of established trig. points may be laid down and made to

assist in the subsequent identification of the points and localities of the sketches.

By a few angular measurements, data for the determination of the position of the trig. points of the primary triangulation by computation through other triangles from the *base line* will be obtained. In the triangles through which the computation may be conducted (which should be by independent routes) a perfect check on the computation and measurement of the angles is secured, so that any error in the determination of the position of the few trig. points of the grand primary triangulation—points checked in at least two series of triangulations—shall be practically a minimum. Within the district embraced in this triangulation the resulting undetectable errors of area or of position are confined to the minimum error, computed in the grand primary or highest triangulation. The check on the results thus obtained should be carried out through all the other triangulations, even to that for the survey of detail.

A survey made by French concessioners under the Spanish government in 1862 may be taken as an example of the French practice in trigonometrical surveys. In this survey the triangles were much smaller than detail triangles on the English system for a similar district. The triangles were equiangular, or nearly so. The size of the triangles ensured the practicability of measuring the angles, but the position of the trig. point was independent of the surface configuration, so that a very important practical consideration was disregarded. The whole of the higher series of triangulations for limiting, detecting, and so abating the resulting errors due to errors of observation and calculation, a feature of great importance in the English trigonometrical survey, is wanting in this survey. The small size of the

independent triangles, and their great number, makes the triangulation for a large district more remarkable for the great number of the triangles than for any principle for detecting, diminishing, or removing the errors which may be fallen into in the numerous independent measurements and calculations.

In some localities the needless multiplication of trig. points was very striking, whilst the position of others was rather unfavourable for angular measurements; and as no check was applied for the verification of the position of remote points, the *building* character of the triangulation was very remarkable. From the number of trig. points selected, their permanent marking would be a work of considerable labour to little purpose, as the number of trig. point objects observed from any one trig. point was limited to those having reference lines intersecting in the point.

In all cases of 'building triangulations' the liability to errors of Observation with the small theodolite used for measuring the angles, and to errors of Computation and Plotting, increases with the number of trig. points. These errors are to a great extent inelimitable, and hence the unsatisfactory character of the results obtained. It has been urged in favour of this system, that these errors ultimately compensate, but this is at best a bad reason for allowing the determination of many points having an unascertained error to stand, that a few undistinguishable points may be ultimately determined free from error. Any advantages this system may have will be best developed in surveys for small districts and close detail in low situations.

Trigonometrical Survey.—The Measurement of Angles.

[The reader who is not already acquainted with the use and adjustment of angular instruments used in surveying, should make himself fully acquainted with what is given on those instruments in Chap. VIII.]

For the purpose of measuring horizontal angles, the theodolite is preferable to the other instruments, particularly for trigonometrical surveys. The theodolite or small transit instrument is superior to the other instruments for this service, because of its optical arrangements, and the greater degree of accuracy with which angles may be measured with it.

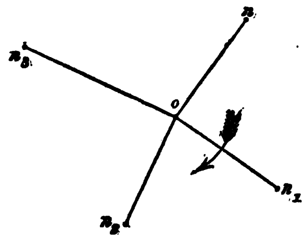
Horizontal angles may be read, with sufficient accuracy for most practical purposes, on a 24-inch horizontal limb for triangles of about 40 miles on the side, on a 12-inch limb for triangles of about 16 to 20 miles on the side, on a 7-inch limb for triangles of about 4 to 5 miles on the side, if the verniers be made to embrace a sufficient number of divisions of the circle, and on a 5-inch limb for triangles of about 1½ to 2 miles on the side. The instruments should be of perfect construction, and furnished with superior telescopes.

For the purpose of measuring the angles at a trig. or other angular point, the theodolite should be adjusted in the vertical over the trig. or angular point, by suspending a conical plummet, point downwards, from a hook for the purpose placed in the stand under the vertical axis. In this adjustment care should be taken to protect the plummet from disturbance by the wind, &c. A small bullet let fall from under the vertical axis will show the point on the ground over which the instrument

shall have been placed with sufficient accuracy for most survey purposes.

Let o be the angular point over which the theodolite is adjusted, and let n, n_1, n_2, n_3, be remote points in the lines intersected in o.

Fig. 20.



The theodolite, when adjusted over o, should be adjusted for observation, all the instrumental adjustments being previously made perfect. The telescope should be directed to the object on the point n, and both plates of the horizontal limb clamped. The vertical limb should be also clamped. The object at n should be finally observed by operating on the slow-motion screws to the horizontal and vertical limbs until a particular point in the axis of the object is apparently covered (bisected) by the intersection of cross wires of the diaphragm. The object or point n is said to be then observed. When the object is observed, the verniers to the horizonal and vertical limbs should be read, and the reading legibly entered in their respective columns on the 'field-book of angles.' The bisection of the object should be again examined, and the reading, entry, &c., checked before disturbing the instrument or unclamping the limbs. The verification being satisfactorily made, the vertical limb and the upper or vernier plate of the horizontal limb should be unclamped, so that that part of the horizontal limb may be made to revolve or turn round the vertical axis without disturbing the lower plate of the limb, which should remain clamped. The telescope should be next directed to the right-hand object n_1, by turning carefully the unclamped

part of the instrument on the vertical axis. The free clamps should be made tight, and the object of the point n_1 observed by operating on the slow-motion screws to the vertical limb and upper plate of the horizontal limb. The verniers should be read, and the readings entered at the point n_1, as already directed for the point n. After completing the verification of the bisection, readings, and entries for n_1, the same parts should be again unclamped; the instrument should be again carefully turned on its axis, and the telescope directed to n_2. The clamping of the unclamped parts, the bisection of the n_2 object, the readings, entries, verification, and unclamping of the same parts should be carefully gone through, for this and every other point observed from o, in the order and manner stated for the point n, until the point n be again observed. The readings for the second observation of the point n should be found to be identical with those for the first observation if the clamp of the lower plate of the horizontal limb and the stand of the instrument be not disturbed during the measurement (on a particular arc) of the angles at o. If the clamp or stand may have been disturbed, the entries on the 'field-book of angles' should be *cancelled*, and the angles re-measured.

In order to remove errors of observation and imperfect graduation it will be necessary, if a high degree of accuracy be desired, to repeat the measurement of the angles on different parts of the horizontal limb. This may be conveniently done, without disturbing the theodolite, as follows:—After the object n shall be observed the second time on completing the *first arc* or angles, the lower plate should be unclamped and the instrument turned on its vertical axis through any angle less than 360°. Then the lower plate should be clamped for the measure-

ment of the angles on the *second arc.* The upper plate should be now unclamped before directing the telescope to the object n. The bisection, reading, and entry should be made by working on the same clamps and slow-motion screws as before directed. The other points, n_1, n_2, &c., should be observed and the readings for the angles entered and verified in like manner on the second as on the first arc. The angles may be measured on other arcs by the repetition of the operations just described.

[It may be seen, by an inspection of the field-book of angles, that the primary and secondary divisions of the graduated limb are entered for one vernier only (No. 1), and that the vernier subdivisions of the secondary divisions are entered for all the verniers. This arrangement is designed to save time in the field by omitting unnecessary entries.]

The difference between the readings for two trig. points, or points on the same arc and vernier, will be the measure of the angle contained between straight lines from these points to intersect in the axis of the theodolite or the angular point. The mean of the readings of the verniers for a particular observation will be the true reading for that observation; and the difference of such true readings for any two particular observations on the same arc will be the measure of the contained angle on that arc. Again, the mean of the contained angle on several arcs will be the true contained angle freed from errors of observation and graduation. It should be found that the sum of the true or correct angles at an angular point shall be equal to 360°. In addition to this check on the measurement of the angles at an angular point, it should be also found that the sum of the angles of each of the plane triangles shall

be 180°.* In some cases there will appear a very small difference between the sum of the measured angles, of a plane triangle, and 180°. This difference may be + or –, and be due to errors in measuring the angles, not altogether removed by measuring them on different arcs; one-third this error should be applied, with the sign + or –, as the case may be, to each of the angles.

In large triangles in which the sides are arcs of great circles, and when the angles are measured without error, the sum of the internal angles will exceed 180°. The excess above 180° is called the spherical excess. To free the spherical excess of errors of observation, &c., it is advisable to take a side of the triangle and its adjacent angles and compute the area as for a plane triangle. For this purpose let A represent the area and r the radius of the earth. Then we have $\frac{A}{r} = \frac{e\pi}{180^\circ}$. In this formula e is the spherical excess and π the circumference of a semicircle. Hence $e = \frac{A \times 180^\circ}{\pi r}$. Here πr is the area of a great circle to the radius r. Again, putting a to represent the area of this circle, we have $e \left(= \frac{A \times 180^\circ}{\pi r} \right) = \frac{A}{a} \times 180^\circ$. From this we find the remaining angle $A' = 180^\circ + e - (B' + C')$. The difference between the computed and measured angle will be the error due to observation. This difference should be apportioned as above directed.

To find the arcs or angles corresponding to the sides of the triangles $a\,b\,c$ (the length of the side a and the

* This is a particular case of the well-known geometrical cannon —viz., the sum of the internal angles of any plane figure, together with four right angles, is equal to twice as many right angles as the figure has sides.

angles of the triangle being known), the following computation should be made:—

Let $A'B'C'$ be the plane angles of the triderial solid angle, at the centre of the earth, corresponding to the triangle. Then from trigonometry $\sin A' = \frac{a}{r} - \frac{a\,3}{6r\,3}$, &c. This gives A' when r is known. And $r = \frac{6a}{6\sin A' + \sin^3 A}$.

[The results from this formula will be sufficiently accurate for most practical purposes.]

To find $b\,c$,

(1) $\log\tan\frac{1}{2}(b+c) = \log\tan\frac{1}{2}a + \log\cos\frac{1}{2}(A-B) - \log\cos\frac{1}{2}(A+B)$.

(2) $\log\tan\frac{1}{2}(b-c) = \log\tan\frac{1}{2}a + \log\sin\frac{1}{2}(A-B) - \log\sin\frac{1}{2}(A+B)$.

By taking the sum and difference of (1) and (2) we find b and c. Then to find the angle A,

$$\tan\tfrac{1}{2}A = \sqrt{\frac{\sin(s-b)\sin(s-c)\,r^2}{\sin s \times \sin(s-a)}}.$$

And to find the spherical excess e,

$$\tan\tfrac{1}{4}e = \sqrt{\tan\tfrac{1}{2}s\tan\tfrac{1}{2}(s-a)\tan\tfrac{1}{2}(s-b)\tan\tfrac{1}{2}(s-c)}.$$

If f be the sum of the angles of a spherical polygon divided into n triangles, having a common angular point, then,

$$e = f - \overline{n-2} \times 180^\circ.$$

From this and the foregoing it may be seen that the spherical excess is proportional to the area, independently of the number of triangles (n) or the number of sides of the spherical surface. Hence equal spherical areas have their excess equal.

The following is an approved form for Field-book of Angles:—

EXAMPLE PAGE OF FIELD-BOOK OF ANGLES.

District of **Antrim.**
Trigonometrical point **Divis Mountain top.**
Verification of trig. point **2-inch hole in flat stone, 4 ft. × 3 ft. 6 in.**
Index error vertical limb **+ 0° 0′ 12″ on D.**

Date **May 6, 1860.**
Observer
Assistants
Theodolite **7-inch, with three verniers.**

Trigonometrical point Observed.	Readings for Horizontal Angles. Vernier No. 1.	2.	3.	Mean of Readings.	Contained Angles (First Arc).	Vertical Angles.	Elevation, E. Depression, D.	Correct Vertical Angles.	Correct Horizontal Angles; mean 1, 2, &c., arcs.	Time of Observation.	Remarks.
	° ′ ″	′ ″	′ ″	° ′ ″	° ′ ″	° ′ ″		° ′ ″	° ′ ″	h. m.	
First Arc											
Black Hill	26 7 14	– 13	– 13	26 7 13·3	98 27 13·3	14 3 20	D.			10.30 a.m.	Top of pole (wicker ball)
Dunrod Hill	124 34 27	– 26	– 27	124 34 26·6	7 28 28·7	5 14 42	D.			11.0 „	(Crossboard) top of pole, clear
Armstrong's Hill . .	132 2 56	– 55	– 55	132 2 55·3	57 44 46	12 3 17	D.			12.20 p.m.	Top of pile, clear, cloudy
Castle Upton Church .	189 47 2	– 1	– 1	189 47 41·3	34 19 4·3	3 6 58	D.			1.10 „	Top of church steeple, cloudy
Molock.	224 6 46	– 45	– 46	224 6 45·6	69 41 2·7	2 20 0	D.			2.25 „	Top cornice chimney shaft, do.
Old Park	293 47 49	– 48	– 48	293 47 48·3	32 24 13	4 30 26	D.			3.0 „	Top of pile, do.
Ormeau Demesne . .	826 12 2	– 1	– 1	326 12 1·3	26 42 11·3	2 12 40	D.			3.45 „	(Crossboard) top of pole, do.
New Forge	2 54 13	– 12	– 13	2 54 12·6	23 13 0·7	2 16 20	D.			4.15 „	Top of pile, clearing fr. clouds
Black Hill.	26 7 14	– 13	– 13	26 7 13·3		14 3 20	D.			5.0 „	Top of pole (wicker bl.), cloudy
					360 0 0						

In order to judge the cause of error in the readings or observations, it is desirable to place on the 'field-book of angles' a statement of the state of the weather and the time of observation, and also a brief description of the form and colour of the observed object. Objects of considerable lateral dimensions, such as towers, steeples, &c., when under a strong side light are liable to be inaccurately observed, the part under strong light appearing greater, and the part in the shade less, than when the whole is under a uniform light.

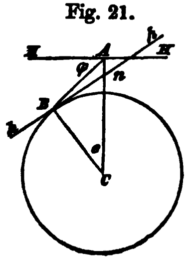
The vertical angles are of less importance than the horizontal angles; they are also much less accurately obtained by observation, and less subject to verification. These angles are distinguished, according as the observed object is above or below the level of the observer, into angles of *elevation* and *depression*. The index error should be placed on the 'field-book of angles' and applied to the vertical angles + or —, as the case may be. The angles of elevation and depression require to be farther corrected for refraction, which causes an object to appear higher than it really is. Hence, for angles of elevation and depression, the correction for refraction is — and + respectively.

The deviation of the ray from the direct line is caused by the difference of density in the parts of the atmosphere traversed and the obliquity of the traversing ray. The difference of density in the atmospheric or gaseous mass is chiefly caused by gravity, the force of which is inversely proportional to the distance from the centre of the earth. The gaseous constitution of the atmosphere renders it subject to local variations of density, due to the development of local heat, which rarefies the part heated. From this it may be seen that the path of a ray near the earth's surface, where local heat is very irregu-

larly generated and evolved, may be subject to many irregular disturbing causes, which render corrections for refraction unreliable. Hence the results obtained by computation from the vertical angles must be regarded as approximations only.

When the trig. points are a considerable distance apart it will be found, after allowing for refraction, that the angle of depression, measured at the more elevated point, will be greater than the angle of elevation taken at the lower trig. point. If the distance between the trig. points be known, the difference in altitude may be approximately found by computation, as follows:—

Fig. 21.



Let A (fig. 21) be the more elevated trig. point.

B the lower trig. point.

ϕ the angle of depression.

θ the angle of elevation.

C the centre of the earth.

e the angle at C. ($= \phi - \theta$).

H the horizon of A.

and h the horizon of B.

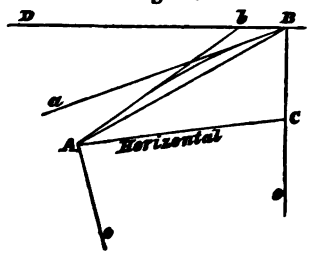
Then $B n \cot e = B C$, the distance of B from C, and $\frac{\cos \theta}{\cos \phi} \times CB = A C$, the distance of A from C. From these we find ($A C - B C$) the difference of altitude—the altitude of A above B.

In those cases where the horizontal distance shall be inconsiderable, the atmospheric refraction may be disregarded for most practical purposes. In cases, however, in which it may be desirable to estimate the effect of local atmospheric refraction for points near or on the

surface of the earth, the following mode of making the estimate may be adopted:—

Let A and B (fig. 22) be two trig. points, and $C A b$, $D B a$, the measured angles of elevation and depression at A and B. The curved line $A B$ represents the path of the ray to which $A b$, $B a$ are tangents at the points A and B, and $A B$ the direct line between these points. Then $C A B$ and $D B A$ are the true angles of elevation and depression, and $A B a$ $(= b A B)$ the error on those angles due to atmospheric refraction. Let $A o$, $B o$, be the direct lines to the centre of the earth from the points A and B, and let $A C$ and R $(= A o)$ be known. Also let the angle $A o B$, subtending $A C$, be represented by Z. Then $C A B + Z = D B A$. Putting r $(= A B a)$ for the error due to refraction, we have $r = \frac{1}{2} (C A b + Z - D B a)$ the refraction at the time of observation.

Fig. 22.



MERIDIAN ANGLES.—*Prismatic Compass.*

The angle on a horizontal plane contained between any two straight lines inclined to one another in that plane, may be found by measuring independently the angle each line makes with the meridian. This may be conveniently done by means of the miner's dial, or prismatic compass. The difference between the magnetic readings—readings on the card attached to the magnetic needle, or meridian pointer, in the box of the instrument—for the lines, is the contained angle, provided the card be graduated from 0 to 360°, and also

provided the difference be less than 180°. If the difference be greater than 180°, its supplement to 360° will be the contained angle. In those instruments where the graduations are numbered from berths, poles, or points, of the magnetic bar, or needle, care should be taken to find the angle by the proper reductions. If the readings have the same zero, and be on opposite sides of the meridian, their *sum* will be the contained angle. But if the readings be on the same side of the meridian, and have the same zero, their *difference* will be the contained angle. Again, if the readings be on the same side of the meridian, but have not the same zero, then 180° less their sum will be the contained angle. And if the readings have not the same zero, and be on opposite sides of the meridian, the supplement of the greater to 180° should be added to the lesser to find the contained angle.

The compass should be held horizontal, or nearly so, that the card or needle may have free motion on the suspension point when in the vertical plane of the line, the horizontal angle between which and the plane of the meridian is to be measured. The eye should be placed to the eye-hole or slit at the back of the glass prism, and the bisection of a distant object 'in the line' made by the vertical wire of the sight vane, held vertical, the prism being so adjusted that the wire and graduation may be seen in contact at the instant of observation. The reading of the graduations will be the magnetic angle or 'bearing.' Before taking the reading the card should have ceased to oscillate. If from any vibratory cause the card will not completely cease to vibrate, then the mean of the extreme readings will be the true reading. [The extreme readings should be entered on the field-book, and not their mean.]

If there be local disturbing forces acting, such as iron

or other highly magnetic bodies, the measured angle will not be the true angle.

The prismatic compass may be conveniently used for laying off angles, tracing lines, erecting perpendiculars, &c.

MEASURING ANGLES.—*The Box Sextant.*

The 'box sextant' is for some survey purposes, particularly for hydrographical surveys, a very convenient instrument. If, however, the district be such that the plane of the observer and the observed points, in lines containing the measured angle, makes a considerable angle with the horizontal, it is not advisable to measure the angle with the box sextant or sextant. Unlike the prismatic compass, the box sextant must be held vertically over the angular point. A distant object in one of the lines should be viewed directly, and a like object in the other line by reflection. When these objects apparently coincide, the reading of the graduated arc will be the contained angle. The measured angle should be corrected for index error, if any, and for parallax if the observed object be very near. For most survey purposes in which the sextant may be used the latter error may be altogether disregarded.

In reading the vernier of the sextant, as in reading other verniers, particularly for graduated arcs, the light should be thrown on, or excluded from, the graduations, so as to illuminate them equally, and so facilitate the correct reading of the graduations. Much of the errors of observation is due to inaccurate reading of the verniers.

Vertical angles may be measured with the box sextant, but for the purpose an artificial horizon is required,* in

* Artificial horizons are made and sold by mathematical instrument makers.

order to obtain reflected images of the object, both from the index glass and the artificial horizon. The angle read on the face of the instrument will be double the angle of elevation. Except for the purpose of observing the altitude of stars or other heavenly bodies, the sextant is not a convenient instrument for measuring vertical angles.

MEASURING ANGLES.—*General Field Work.*

The skeleton map or diagram of established trig. points should indicate, besides the position of the trig. points, the leading surface configuration. This latter may be conveniently supplied by shading or etching on the skeleton map, which should give the angles for plotting or laying down the position of the trig. points. These points should be distinguished by names or characters, which should be also written on their respective sketches, with the measurements to neighbouring detail, for the purpose of refinding the trig. points. Both the sketch map and the detail sketches with measurements should be supplied to the surveyor measuring the angles, in order that he may verify his position at any trig. point, and enter the proper distinguishing name or character to each observed point.

The angles of each series of triangles should be measured with theodolites of the same and sufficient size and power for the purposes of the survey. All the points observable from a particular trig. point should be taken up in regular order, if convenient, and the readings taken on several arcs, as already explained. If convenient, the observations should be made, particularly for the 'grand' and 'primary' triangulations, not only on the same day on different arcs, but also on different

days and in different states of the weather. For the trig. points of the smaller triangles less numerous observations will suffice.

In the 'grand,' or other triangulations, remote trig. points (beyond the range of the power of the telescope at the time of observation) may be observed by the reflection of the sun's rays or other strong light from the remote point to the station of the observer. This may be conveniently done as follows:—

Near the remote point, but distant from it about four chains, in the line of observation between this point and the station of the observer, a ranging rod should be set up, on which a polished metal ring should be so suspended that the ray to the observer may pass approximately through it. At this trig. point a reflector, mounted on a frame with a stiff universal joint, should be adjusted at short intervals of time by a party for the purpose, that the sun's reflected ray may be thrown on the polished metallic ring. This should be continued until it be reported that the point is sufficiently observed. At the same time that the reflected ray is being thrown on the metallic ring, the party measuring the angles —observing trig. points—should point the telescope, of a properly adjusted theodolite, to the remote trig. point; and in observing keep cautiously moving it on a small field, by operating on the slow-motion screws to the upper plate and vertical limb (the whole of the members being clamped) until a flash of light from the reflector enters the telescope. In this position the instrument should be allowed to remain, with the eye still kept to the telescope, until a second or third flash be observed. This will verify the bisection of the remote trig. point reflector object.

In the triangulations for the smaller triangles the

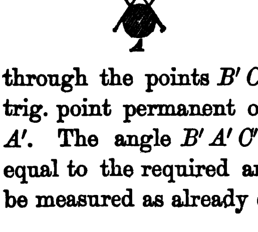
poles, or temporary trig. point objects, should be removed, to allow the theodolite to be accurately adjusted over the trig. point. After the angles are measured at a point and a theodolite removed, the pole or other object should be carefully replaced and secured in the vertical position.

Sometimes it may be convenient to select a permanent conspicuous object, such as a tower or church steeple, for a trig. point. In these cases the axis of the tower or steeple will be the vertical of the trig. point, in which it will not be convenient to place the theodolite to measure the angles at that point. The angles at such inaccessible trig. points may be found as follows:—

First Method, by equal Angles.

Let A (fig. 23) be the inaccessible trig. point, B and C remote trig. points. It is required to find the angle $B A C$. At B and C distances $B B'$, $C C'$, each equal to the radius of the trig. point, permanent objects should be laid off perpendicular to the lines $B A$, $C A$ at those points; and on the points so determined suitable objects for observation should be erected. Lines

Fig. 23.

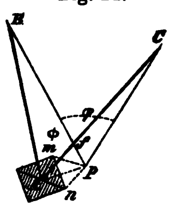


through the points $B' C'$ should be traced to touch the trig. point permanent objects and intersect in the point A'. The angle $B' A' C'$ at A', an accessible point, is equal to the required angle $B A C$. The angle A' may be measured as already described.

Second Method, by an Auxiliary Angle.

Let A (fig. 24) be the inaccessible trig. point, BAC the required angle, B and C being remote trig. points. An auxiliary point p should be selected near A, from which B and C may be observed. The lines mp, np, mn, and the angle mpn, should be measured. The radius nA of the circumscribed circle and the angle mnA is supposed to be known, so that the distance Ap may be found by computation. The computed approximate lengths of AB, AC should be also known. At p the angles made by the lines from m, B, and C should be measured. These complete the data for finding the angle A by computation.

Fig. 24.



Putting r to represent Ap

	ϕ	,,	the angle BpC
	Φ	,,	the angle A
	y	,,	the angle $BpA (= mpB + mpA)$
	a	,,	the line AC
and	b	,,	the line AB. Then

$$\Phi = \phi + pCf - ABf. \qquad \text{And}$$

$$\sin ACp = \frac{r \sin(\phi + y)}{a}. \qquad \text{Also}$$

$$\sin ABp = \frac{r \sin y}{b}.$$

The angles found by their sines in these equations being very small, the sines and arcs may be taken as equal, without serious error. Then we have from above—$\Phi - \phi = pCf - ABf$, and by substitution

$$\Phi - \phi = \frac{r \sin(\phi + y)}{a} - \frac{r \sin y}{b}.$$

The quantities on the right hand side of this equation are all known, hence $\Phi - \phi$ may be found. From this we have $\Phi - \phi + B p C = A$, the required angle.

It may be observed that a, in the formula, is the line to the right, and b that to the left; and if p be on the left side of $A C$, then $\phi + y$ would exceed 180°, in which case the sine would be negative.*

Measurement of the Base Line.

In order to obtain by computation the correct length of the several sides of the triangle of a trigonometrical survey, it is necessary, in addition to the measurement of the angles at the angular points, to find, by very accurate admeasurement, the length of a side of one of the triangles.

If the triangulation be extensive, the lengths of *one* or *more* sides of remote triangles should be found in like manner, so as to verify the computed lengths of these sides. These latter lines are generally called 'bases of verification.'

The selection of the lines for admeasurement is of much importance, for on the accurate determination of their lengths depends the possibility of the accurate determination of the lengths of the sides of the other triangles by computation. It is farther desirable that the computation of the length of each line, or side, may be made on several independent data, so as to detect arithmetical or typographical errors in the computations or in the book of logarithms used, which, if undetected, would give false or erroneous results. From this it may be seen that the *base line* should be as near the centre of gravity of the district as circumstances will admit, and

* Abridged from Puissant's 'Geod.' vol. i. p. 210.

that its extremities should be observable from as wide a range on both sides of it as may be. For the latter reason, and the proper conduct of the computations to a higher series of triangles, this line should be selected by the party selecting the position of the trig. points. But these are not the only conditions and considerations to be attended to: it is also desirable to have regard to the facilities for making accurate measurements along the line between its extreme points. For the latter purpose it is essential that the ground be firm and level, or nearly level. It should be free from surface encumbrance, and if not free, it should be freed from such encumbrance.

If the ground on which the *base line* shall be measured be not level, or a uniform plane, the intersection of the several planes between the extremities of the line, as well as the extreme points, should be marked on permanent objects erected in the trace of the line at these points. The permanent objects may be built of masonry, capped with a suitable cut-stone, having its top surface level. The height of this stone above the ground should be such that the straight lines joining the top surfaces may be above the ground at every part, and be as near as may be to two feet from it throughout, so that the plant on which the measuring instruments shall be supported may have this line in their supporting surface.

The extremities of the base line should be finely marked on the permanent objects erected for the purpose, and the line between these points should be accurately traced with the theodolite, particularly on the intermediate permanent objects, on which the plane points should be finely marked. The ground, if uneven on the surface, should be made smooth and free from irregularities.

The plant alluded to should be made of light, well-

seasoned wood, properly put together on a suitable sole-plate provided with adjusting screws, &c., for adjusting the bearing surface in the line.

The measuring instruments may be made of different materials, such as glass, wood, iron, &c. Glass and wood are generally made into rigid wands, and iron into wands, flexible thin lengths, and chains composed of wire links connected by circular or oval rings. The materials which expand least with heat, and may be made in suitable lengths without splicing, are preferable to highly expansive or spliced measuring instruments. The standard temperature for the standard length on a particular material should be known on several thermometers. These should be used in the measuring operations. The errors due to temperature may be computed by knowing the expansion of the measuring wand per degree of heat, and the difference of temperature between that for standard length and each length laid down in measuring the line.

The operations for measuring the *base line* (which shall be presently described) should be performed with great care in every particular. On commencing the measuring operations for the whole or any section of the line, the stands for supporting the measure of length in the line should be adjusted in the line at one end of the section or line. On this stand the measure of length should be extended, and its zero accurately adjusted to the zero of the line. This may be done by clamping the wand or measure to its supporting-stand, and making the adjustment with a suitable slow-motion screw. A second and third measuring wand should be supported and adjusted in the line, and to the first and second measuring wands in the above manner, before disturbing the first stand and measuring wand laid

down. The temperature of the several wands laid down in the line should be observed on thermometers, at different points in each, and the adjustments verified before removing the stand No. 1 to the forward part of No. 3 stand (if three stands only be used), or to the forward end of the last stand adjusted in the line. The measuring wands should be distinguished, and the temperature for each entered to the description of its proper wand, &c. After the wand is brought forward and re-adjusted in the line, the like observations and re-adjustments should be again repeated and carefully noted before bringing forward the rear stand to the front. In this manner the measurements should be made for the whole and every section of the line.

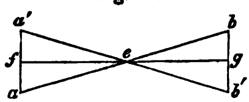
If the measuring instrument be flexible or jointed, such as a steel tape line or chain, it should be drawn out with a given standard force by placing a weight over a pulley, and attaching it with a wire to the end of the measure. In this case the temperature should be also taken at several known points on the measure for each length laid down. The base line in the several planes should be several times measured in this manner, and a mean taken of the reduced lengths to find the correct measured length.

The measured length found by the direct measurements should be corrected for temperature by computing for the several parts of each wand or measure the error due to difference of temperature, as already explained. The difference between the sums of the plus and minus errors should be added to or taken from the measured distance as the case may be, to find the correct or standard length of the line or section.

It may be expected that in laying down the numerous lengths, and making the still more numerous observa-

tions of temperature, some slight errors will be found to slightly vitiate the reduced or standard length. In order to reduce the liability to error by dispensing with the thermometric measurements, the officers of the Royal Engineers have brought into use, in the measurement of the *base line* for the trigonometrical survey of Great Britain, Ireland, and the Channel Islands, a very ingenious and perfect measuring instrument, which measured the standard length under every change of temperature in these latitudes. The following brief description of this instrument will suffice here. Two bars of metal $a\,b$, $a'\,b'$ (fig. 25), of known expansion, are attached to one another at the middle e, and at their extremities to two other bars $b\,b'$, $a\,x'$, whose expansion is known to be equal. The rate of expansion being known for each bar, the length of the bars $a\,e$, $a'\,e$, and $a\,a'$ may be found by computation, so that a point f in $a\,a'$ shall be always a constant distance ($f\,e$) from the centre point under equal changes of temperature in those bars. The line $f\,g$ will therefore be an unchanging measure of length for all temperatures.*

Fig. 25.



If the section or line measured be horizontal, the mean of the reduced or corrected measured distance will be the true distance. If, however, the plane be inclined, its measured length, corrected for temperature, should be farther corrected for inclination to find the true horizontal length. To make this correction, the difference of level between the bench marks, or plane points, for each plane and section should be found by

* Much interesting information will be found on this subject in a work on the Ordnance Survey Base, &c., by Captain Yolland, R.E.

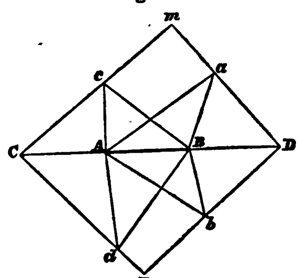
accurately levelling between these points several times, and taking a mean for the true difference of level. From these measurements—the length of the incline and vertical height—the corresponding horizontal length may be found by computation. This computation may be arithmetically made by extracting the square root of the difference of the squares of the inclined length and vertical height, which will give the required or true distance for that section or line. If the *base line* contains several sections or planes, the sum of the true horizontal lengths of these sections will give the true length of the *base line.*

Prolongation of a Measured Base.

If the extent of suitable ground be not sufficient for measuring directly the whole of the *base line*, its length may be found by the admeasurement of an intermediate part or section as follows:—

Let $A B$ (fig. 26) be the measured section, C and D the trig. point extremities of the *base line*, m and n other trig. points commanding a view of C and D. The lines from m and n to C and D should be accurately traced at $c\,a$ and $d\,b$, points from which A and B shall be visible. At the points A and B the intermediate points a c, d b should be observed, and also the points m n, if practicable. The theodolite should be placed on the intermediate points, from which the extreme points of the

Fig. 26.



measured section of the base and *base line* should be observed. The angle at an intermediate point subtended by $A B$ should not be less than 30°. It is assumed that the angles at the trig. points $D m$ $C n$ are measured. Then, from trigonometry, we have in the triangle $A B a$

$$B a = \frac{A B \sin A}{\sin a}.$$

In like manner $B D$ may be computed in the triangles $B a D$ and $B b D$, and $A C$ in the triangles $A d C$ and $A c C$. These computations give the verification of the distances $C A$, $B D$, and hence the length of the *base line* by adding the measured section $A B$. The section $A B$ should not be a small fraction of $C D$.

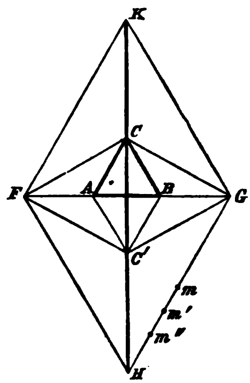
As a farther, but indirect, verification of the ascertained length of the base line, by the addition of the computed and measured parts or sections, the common side $C m$ should be computed in the triangles $C m D$, $C m B$. In like manner the other sides of the triangles $C m D$, $D n C$, adjacent to the *base*, should be computed with the whole *base line* (consisting of *one* measured and *two* computed sections) and the measured and one computed section as the given side.

Combined Measured Bases.

The difficulty of finding a suitable locality for measuring a *base line* of sufficient length for the purposes of the survey, may make it important to select a locality suitable for the determination of the *base line* from *combined measured bases.* These may be much more expeditiously and conveniently measured, and the determination had, than in the mode just described for finding the length of the base line. The mode of proceeding is as follows:—

On suitable ground or level plain, if even of small extent, a comparatively short base line $A\,B$ (fig. 27) should be carefully and accurately measured, as already explained, with the most approved apparatus, and its true length very accurately found. The extremities at A and B should be previously marked on permanent objects, as already described. At these points the equal angles $C\,A\,B$, $C\,B\,A$ should be laid off, each the angle of an equilateral triangle, and the lines traced to meet in C. At the point of meeting a third permanent object should be erected, and the point of intersection finely marked on it. The sides $A\,B$, $B\,C$ of this equilateral triangle should be also found, by equally accurate admeasurement, to verify the rigid accuracy of the construction of the equilateral triangle $A\,B\,C$, or 'combined measured bases.'

Fig. 27.



From A and B angles of an equiangular triangle should be laid off on the other side of $A\,B$, and the lines traced to intersect in the point C'. This may be very conveniently done by laying off with two theodolites the angles, and tracing the lines at the same time from both the points A and B, so that the axis of the object on the point of intersection may be bisected, at the same instant, by observation from both stations of the observers. From the latter determined point and C the angles of equiangular triangles ($F\,C\,C'$, $G\,C\,C'$) should be laid off on opposite sides of $C\,C$, and traced as described above, or in some other convenient manner, to intersect in the parts

F and G. If the angles be accurately laid off, and the lines properly traced, the points of intersection F and G should be found to be in the trace, or prolongation, of AB. And from the latter determined points, angles of the equiangular triangles should be laid off, and their lines traced as above to intersect, on opposite sides of the line FG, in the points HK, which should be found to be points in the line CC' produced. In this manner the *base line* or *base lines* may be extended in the lines AB, CC' produced as may be required, and the determination of the point verified by the instrumental measurements.

In tracing long lines over broken country it may not be practicable to command from the distant points a view of the point of intersection of the lines laid down in laying off the angles at those points. In those cases the lines should be traced forward at intermediate points, such as m, m', m'', H, in the manner hereinafter described for tracing lines with the theodolite.

The lengths of the lines may be found as follows:—

$$CC' = AB\sqrt{3},$$
$$FG = AB\sqrt{9},$$
$$HK = AB\ \overline{27}, \text{ and generally}$$
$$CC'_n = AB\sqrt{3_n}.$$

A Suggestion for Practical Solution.—Measurement of Base Lines.

Before disposing of this important subject, it may be well to offer a suggestion which may be found valuable for certain localities, and offer special advantages in others.

The chief difficulty in finding a suitable locality for

the measurement of a base line arises more from the yielding nature of the turf covering of the firm ground than from inclination or surface irregularities. If a means of measurement be adopted which shall not require the surface of the ground to be firm, the difficulty in finding a suitable locality for the base line will be greatly lessened if not entirely overcome, as bogs, or peat moss, the level parts of the valleys of large rivers, fens, and even lakes, may be found sufficiently extensive and otherwise suitable for a *base line.*

In the preceding operations distance is measured by a line of given length, and hence the necessity for suitable ground on which to lay down the several lengths of the line of length or measuring wand. Distance may be also found by a measurement of quantity, if the quantity be uniformly or proportionally distributed over the distance. Such a measure of distance, if properly applied, would satisfy all the requirements in most localities. The uniform distribution of material may be effected in various ways, and the material may be either solid or liquid. We shall suggest the application of both, in a manner to bring out a verification of the measurement of a *base line* in a locality unsuitable for the direct measurement of distance by any other means. The locality selected may be a lake, lagoon, or other still-water surface. The means for making the measurement may be a water-tight steel tube, of small uniform bore and of the full length of the base line. This tube may be enclosed in a uniform gutta percha coating surrounded by light material, so as, when filled with water, to float it at all parts at the water surface of the lagoon or lake. Through this tube, at the time of fitting and soldering the parts to one another, a uniform steel wire of the full length should

be passed. The tube may be passed across the lagoon by a boat's crew, from one end of the base line to the other, to the vicinity of the trig. points. The tube will be visible at the surface, and so its position in the straight line may be verified by a trace of the *base line* with the theodolite. When properly extended and adjusted in the line, the ends of the tube and the internal wire should be attached to apparatus for keeping the tube and wire so extended, and for conveying water into the tube at one end and discharging it at the other, so as to expel the air and completely fill the tube with water. This may be, perhaps, more conveniently done by holding the forward end of the tube a few feet under water in extending it across the lagoon or lake. If the boat be rowed at a low rate of speed, the strain on the pipe by the in-flowing water and out-going air will be inappreciable. The temperature of the water in the tube and also of the iron wire should be observed at the ends and known intermediate points, at which extreme and intermediate points suitable valve appliances should be inserted, for making a gauging of the contents of the whole length at one or more operations. The water contents of the tube may be drawn off, and its weight or measure verified by taking the temperature and a direct gauging. The enclosed wire should be withdrawn in suitable sections, and, after being dried, it should be weighed and temperature taken. After the wire shall be withdrawn the tube should be refilled with water, and the observations for temperature made as before. It may then be discharged, and the contents verified by gauging, &c. These measurements should be repeated several times before and after the withdrawal of the wire, and a mean taken of the series or sets of measurements, for the correct

gauging or contents of each. The difference in the gaugings will show the quantity of steel wire displacement. If the displacement at a known temperature for (say twenty) equal pieces of the steel wire, of an accurately ascertained length, be measured, sufficient data will be obtained to compute the distance, the law of expansion of water by increase of temperature being known. This will be checked by taking the quantity in terms of the ascertained weight thus:—

Let W be the weight of the displaced water, discharged so as not to suffer loss by evaporation, and let W' be the weight of the displacement for a length of wire (say) $20\,l$. Hence weight of displacement for l, the accurately ascertained length, will be $\frac{W'}{20}$. From these data we have—

$$L = \frac{W}{W'} \times 20\,l,$$

the total distance between the ends of the tube, if the temperature of the wire for the determination of W' and W be equal. If the temperature be not equal, a corresponding correction should be made on l and the whole computed distance to find the correct distance.

Tracing Straight Lines with a Theodolite.

In tracing lines, such as for the prolongation of the measured *base* line, or the laying down the lines of a given angle, as in the determination of the vertex of an equiangular triangle in 'combined measured bases,' the theodolite should be placed and adjusted for observation in the vertical of the angular or other point of the line to be traced. The telescope should be directed to a distant well-defined point in the line, and

the horizontal limb clamped. The bisection of the remote point may be made with the slow-motion screw attached to either plate of the limb. The line of collimation will then be in the vertical plane of the given line, so that points on the surface bisected by the intersection of the cross wire will be in the required trace of that plane or of the line. Let G (fig. 27) be the station of the theodolite, and GH the line to be traced. An assistant, provided with a good field telescope and a hand (conical-shod) flag pole, should place himself approximately in the line at (m) the intermediate point to be determined. He should hold the pole vertical, and direct his telescope so as to bring the observer into its field of view. At the same time the observer should seek for the position of the remote assistant in the field of view of the theodolite telescope, as it is made to travel along the line by turning gently, with the eye to the telescope, the vertical limb on its axis. On observing the position of the remote assistant, if the flag pole be not in the line to be traced, the assistant attending at the station of the theodolite should move laterally to the opposite side of the line to that on which the remote assistant has taken up his position, and set up his flag pole to indicate the direction that assistant should move to come into line. The distance of the signal flag pole from the theodolite should indicate the extent the remote assistant's position is in error. The remote assistant, observing the movements, direction, and probable extent of error thus signalled, should move accordingly and steadily towards the line, halting frequently to erect the flag pole and observe the signal flag pole of the observer's, until, finally, he takes up a point in or beyond the line. Then the observer's flag pole should be lowered. If the remote point be accurately in the

line the signal flag pole should be taken to the immediate rear of the observer, and after remaining up some minutes it should be lowered, to indicate that the remote flag pole point is a point in the line. If, however, the remote point be taken up on the opposite side of the line to that first taken up, the signal flag pole should be taken to the opposite side and erected as before, until, under the direction of the observer and the combined movements of the remote and immediate assistants, the remote flag pole be accurately placed in the line and the proper signal to that effect given. The remote assistant should then mark the point and again set up the flag pole on it, to have the position verified before proceeding to the determination of other intermediate points in the line. Other points, as m', &c., may be determined in like manner. If the point H be not visible from G, the theodolite should be taken to m'; it should be placed in the vertical of this point, and adjusted for observation, as already described. The clamps of the Ys should be opened and the telescope directed to the object G, which should be observed, with the clamps of the horizontal limb tight, and the intermediate points m, &c., again examined if visible from m'. The telescope should be carefully reversed in its Ys, and the trace of the line continued, as above, to H. Before disturbing the instrument the telescope should be again reversed, and the object at G again observed, in order to verify the position and trace of the forward points. The lines CH and FH should be traced in like manner, according to circumstances. The point H will therefore be the point of common intersection of the traced lines.

In tracing secondary reference and other lines, the extremities of which are visible from an intermediate

point, but not visible from one another, the theodolite should be placed approximately in the line, at the intermediate point, and adjusted for observation. One of the end point objects should be observed, and the telescope reversed in its Ys, as already described. By making the vertical limb revolve, it will be observed if the other end, or reference point, object be bisected by the intersection of the cross wires. If the object be 'bisected,' the theodolite will be adjusted in the line; if this be not the case, the theodolite must be placed on a lateral point more nearly in the line, and the same adjustments and operations again gone through at this position of the theodolite. In this manner, by 'trial and error,' the theodolite may be finally brought into the line, and the intermediate point accurately determined. From this point the trace at other intermediate points may be determined by signalling with the hand, &c., without the assistance of a field telescope or special assistants. In signalling with the hand, the upraised extended arm indicates the direction of the movement required. When both arms are upraised and extended, it signals that the point is accurately determined.

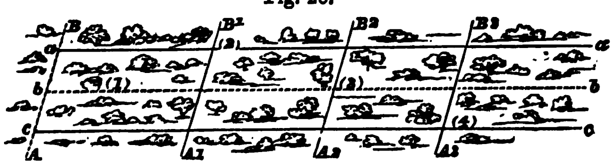
In the foregoing directions for the tracing of straight lines with the theodolite, it was tacitly assumed that the trace was over open country, free from surface encumbrances. In such country, accidental obstructions to the detail trace may be conveniently made by adopting a rectangular or other parallelogram, the opposite sides of which should be measured. The equal angles should be laid off with the theodolite. If the sides of an offset isosceles triangle be measured, the line may be traced past an obstruction by laying off the angles with the theodolite. When, however, the country is generally encumbered, as in plantings and forests, the obstructions

are more numerous and less manageable. In such country some of the following methods may be adopted, according to circumstances.

The line may be traced in forests or plantings, in which there may not be underwood or general surface obstructions, by the aid of auxiliary parallel lines. All the lines should be continuously traced at the same time.

In the illustration (fig. 28) *a b c* are the parallel lines, and *a c* the auxiliary lines. *A B* is a line, or direction, in which *b* is a trig. or reference point. The distances *a b*, *b c* in this line should be measured, and also the angle which *b b* makes with *A B*. The line *b b* and the auxiliary lines should be traced on this angle,

Fig. 28.



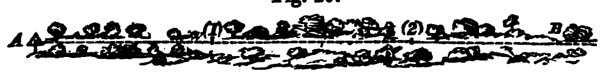
until the trace of one of the parallel lines be obstructed by a tree, such as *b* at (1). Immediately on passing the obstruction on one of the auxiliary lines, a line should be traced on the measured angle or its supplement, as may be required, and traced to intersect the other auxiliary line. The angle made by these lines should be measured at the point of intersection, to verify the trace of the intersecting lines. From these angular points in the auxiliary lines, the distances ab_1, cb_1 should be measured in the transverse line, and found to meet, but not overlie, one another. Then will the point of meeting in the transverse line be a forward point in the line *b*. At a suitable distance forward from which b_1 may be observed, a like determination of another point, b'_1 in *b*,

should be made. The trace of the line *b* should be taken upon these points and continued in connection with the auxiliary lines until the trace of one of the lines be obstructed at some other point, such as *a* at (2). The trace of the obstructed line should be taken up by measurements in the transverse line $B_1 A_1$, and in a forward parallel line, and the traces continued as described above. In like manner the obstructions at (3), (4), &c., may be passed, and the trace of the line continued for considerable distances with sufficient accuracy for most practical purposes. The continued parallelism of the lines at the measured distance apart will be a sufficient verification.

For brushwood or bush districts, in which the surface encumbrance is more stunted and dense, the line may be more conveniently traced as follows:—

At *A* (fig. 29), a principal point, a stage for the

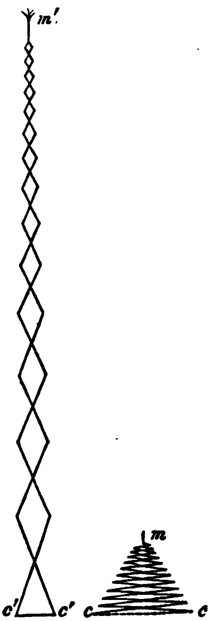
Fig. 29.



observer should be erected at such height as may be required to command a clear view of the over-wood trace of the line. For this purpose it is desirable that the site of *A* be elevated. The theodolite should be set up on the stage, and the trace of the line (*A B*) made in calm weather. The party for marking the determined intermediate points should be provided with a prismatic compass, suitable overwood flag apparatus (see fig. 30), and fine (silk or india-rubber covered) telegraphic wire and wheel-reel. The length of the telegraphic wire should not be less than the length of the line to be traced. For the purpose of tracing the line in the night time a magnesium wire light will be desirable. The overwood

flag apparatus $m' c' c'$ (fig. 30) should be made of light wood, to be portable, and sufficiently strong to bear its own weight when extended, as in the margin. The lower ends of the diagonal bars should be furnished with a screw wire for tightening up the frame and keeping it fully extended. The elevated point m' may be made conspicuous above the vegetation with a red flag by day and the magnesium wire light by night. The observer at A should give to the forward, or detail-tracing party, the magnetic angle of the line to be traced, by which they may keep close to the line. One end of the telegraph wire should be connected with a signalling apparatus at A, and the other with a like apparatus in charge of the tracing party. By this means the observer will be able to direct the movements of the party at the remote intermediate points in the line, until the elevated point m' be accurately adjusted in the line. A plummet from the point will refer the trace to the surface. The point A_1 thus determined should be properly marked, and a survey made of surrounding objects, which should be also specially marked, to aid in a subsequent identification of the point. The approximate distance from A will be given by the length of telegraph wire paid out. In this manner the line may be traced at other intermediate points, for the length of the telegraph wire.

Fig. 30.



After the line is traced to this extent, the travel of the tracing party should be retraced, and the wire re-rolled for use in the prolongation of the trace of this line or the trace of other lines. If suitable intermediate points be identified by conspicuous flag-pole objects, these may be observed from the principal points for observation, which will enable the surveyor to lay down their position on a map from computed (trigonometrical) distances. By this arrangement sections of the lines on which there may not be important detail, or which may be heavily obstructed, may be, in the first instance, and for most purposes in outlying districts, left unsurveyed.

Finding Inaccessible Distances with the Aid of the Theodolite.

In describing the methods for finding inaccessible distances with the chain, Chap. II. (illustration 15), the formula for finding inaccessible distances with the aid of the theodolite is given, and in the foregoing part of this chapter the measurement and laying-off of angles are more particularly described. The angles computed from the formula (see Chap. II.) are given here for convenience of reference. [A copy of this table should be inserted in the back of the detail field-book.]

When $p = 1$,	$\phi =$	$45°$	$0'$	$0''$
„ $= 2$,	„ $=$	26	33	54
„ $= 3$,	„ $=$	18	26	6
„ $= 4$,	„ $=$	14	2	$10\frac{1}{2}$
„ $= 5$,	„ $=$	11	18	35·8
„ $= 6$,	„ $=$	9	27	44·28
„ $= 7$,	„ $=$	8	7	48

When the approximate distance will suffice, as in reconnoitring, these angles may be laid off with a prismatic compass or box sextant.

TRACING LINES WITH THE PRISMATIC COMPASS.

Lines may be traced with this instrument with more rapidity than accuracy. The magnetic angle of the line should be measured, which will give the direction when the compass is on any point in the line. When the compass is held at a forward point in view of a point in the line, the reading should be the measured angle + 180°. The observer in this situation should change his position until it be so taken up, that on observing the known point in the line, the wire of the sight vane shall mark the correct angle on the card. Then will the observer be on a point in the line.

COMPUTATION OF THE SIDES OF TRIANGLES.

The sides of the several triangles should be computed from data obtained from different independent triangles, as already stated. In plane trigonometry it is shown that the sines of the angles of a plane triangle are proportional to the lengths of their opposite sides. From this it may be seen that, if the angles be represented by $A\ B\ C$, and their opposite sides by $a\ b\ c$ respectively, then $a = b \times \frac{\sin A}{\sin B}$, which is the general formula.

In the conduct of a trigonometrical survey it may be desirable to compare the measured and computed extent of an angle, or determine the position of a hanging or other trig. point. In those cases two principal combinations may occur in the given data for the computation. These combinations are :—

I. Two sides and a contained angle.

II. Two angles and a side.

In II. the data may place the angles adjacent to the side or one of them opposite to it.

In I. the unknown parts are a side and its adjacent angles, and in II. the sides and a contained or adjacent angle.

To find the angles adjacent to the unknown side in I. we have from trigonometry:—

$a + b : a - b :: \tan \frac{1}{2}(A + B) : \tan \frac{1}{2}(A - B)$. Taking half the sum and half the difference of the angles in the two last terms of the proportion—the last being the computed term—we have $\frac{1}{2}(A+B) + \frac{1}{2}(A-B) = A$, the greater angle, a being the greater side, and $\frac{1}{2}(A+B) - \frac{1}{2}(A-B) = B$, the lesser unknown angle. The remaining side may be computed from the general formula.

To find the unknown parts in II. we have—

$$\sin(A+B) : c :: \sin A : a;$$

$$\text{or } a = \frac{c \times \sin A}{\sin(A+B)}.\text{*} \quad \text{And}$$

$$\sin C = \frac{c \times \sin A}{a}.$$

The lengths of lines in the detail triangulation computed from these formula should be compared with the measured distances, as a check on the return of field work in the detail content, or other field-book. As already stated, the errors in the measurements of the lines should not exceed 1 in 1,000 in open country, or $1\frac{1}{2}$ in 1,000 in stiffer and hilly country.

When it is required to obtain a map of a line of travel with the least length of lines and in the shortest time,

* There is a particular relation of c and its opposite angle, which would render the determination of a from the formula doubtful. See Chap. VI. (Computations).

or when it is required to find, approximately, the area of a district of moderate extent, a survey by 'traverse' may be adopted. If the lines of this survey make a complete 'close,' or complete the traverse polygon, the enclosed area may be computed. In the traverse the sides and angles of the polygon should be measured, the former to the horizontal with the chain, and the latter with a suitable angular instrument. The detail should be surveyed on the sides of the traverse polygon as reference lines, or on other reference lines, making a known angle with a line of the polygon, which it should intersect at a given or known reference point.

If the angles be measured with a prismatic compass, or, more correctly, the magnetic meridian angles for the sides of the polygon be measured for each reference line of the polygon, it is desirable that this angle be measured by observing a remote point in the line from another point in the same at or near either end. The difference of the angles for each line should be found to be 180°, if the angles be accurately measured.

The traverse may be more or less complete according as the reference lines of the survey do or do not make a polygon. In a complete traverse, or 'close,' the sum of the internal angles added to 360° shall be equal to the product of 180°, multiplied by the number of angular points.

If there be no great difference of level between the extreme angular points of adjacent lines of the traverse polygon, the angles may be conveniently measured with a box sextant. If, however, there be any considerable difference of level between these points, the angle measured being in their plane, the use of this angular instrument is objectionable. The angles measured with the box sextant should be reduced to angles

made with a fixed direction or meridian to obtain the angles suitable for use in the computation for areas.

A more accurate measurement of the angles may be made with the theodolite. The reading of the horizontal limb may be made to give directly the angle made with a fixed direction or meridian, as in the measurements with the prismatic compass, or it may be made to give the angle independently, as in the measurements with the box sextant. The former mode for measuring the angles affords a check on the manipulation of the instrument, and gives more conveniently the data for the computation of areas.

The angles which the sides of a traverse polygon make with a given direction may be measured with the theodolite as follows:—

At the end of the first measured reference line, or the first line of the traverse polygon, the theodolite should be adjusted over the angular point, and also adjusted for observation. The telescope should be directed to an object on the remote end of the first line, which should be observed, the horizontal limb being previously clamped. The verniers should be then read, and the readings entered thus: $\dfrac{47-2'-16''}{227-2\ -16}$. The first vernier gives the angle which the line makes with the zero diameter, which, for convenience, may be taken as the line of direction. The upper plate of the horizontal limb should be unclamped, and the telescope directed to an object at an intermediate or the extreme remote point in the second line, which should be observed, as already directed in the foregoing part of this Chapter on the measurement of horizontal angles. The reading for the second line will give the angle, as above, which this line makes with the fixed line of direction. After verifying the observation and readings at the first

point for the second line, the theodolite should be carefully removed, without shock, with the upper plate securely clamped to the second angular point, over which it should be adjusted for position and observation. The telescope clasps should be carefully and cautiously opened, and the telescope equally cautiously reversed in its Ys. The lower plate of the horizontal limb should be unclamped, and the telescope directed to the object in the second line, on or near the first angular point; the lower plate should be next reclamped, and the object observed by working tho slow-motion screw to the lower plate. The zero diameter of the horizontal limb will be now adjusted parallel to its position at the first angular point. The telescope should be again reversed, the upper plate unclamped, the forward object on the (third) forward line observed, and the readings entered as directed above. This gives the angle made with the line of direction. The angle made by the forward lines with the fixed line of direction at the successive angular points may be measured in like manner. It may be seen that the line first measured (first) will be the forward line for the ultimate or last angular point, and hence the first or backward reading at the first angular point should be identical with the forward reading at the last angular point, the upper plate of the horizontal limb having passed through 360°.

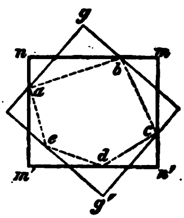
It may be observed that the adjustment for parallelism of the zero diameter of the instrument at the several angular points is made, after the reversion of the telescope, with the clamp and slow-motion screw to the lower plate of the horizontal limb, and that the adjustment for the measurement of the angles in the observations of the forward objects is made with the clamp and slow-motion screw to the upper plate of the limb.

The measurement of the angles, as independent angles, has been already described. If the successive angles be measured without reversing the telescope, by continuous readings on the horizontal limb, the angle made by each line with a fixed line of direction, such as the line of the zero diameter of the first angular point, may be found by deducting 180° from each alternate reading after the first.

The computation for area may be made to afford an approximate check on the relative accuracy attained in the measurement of the sides and angles of the polygon. This check may be obtained as follows:—

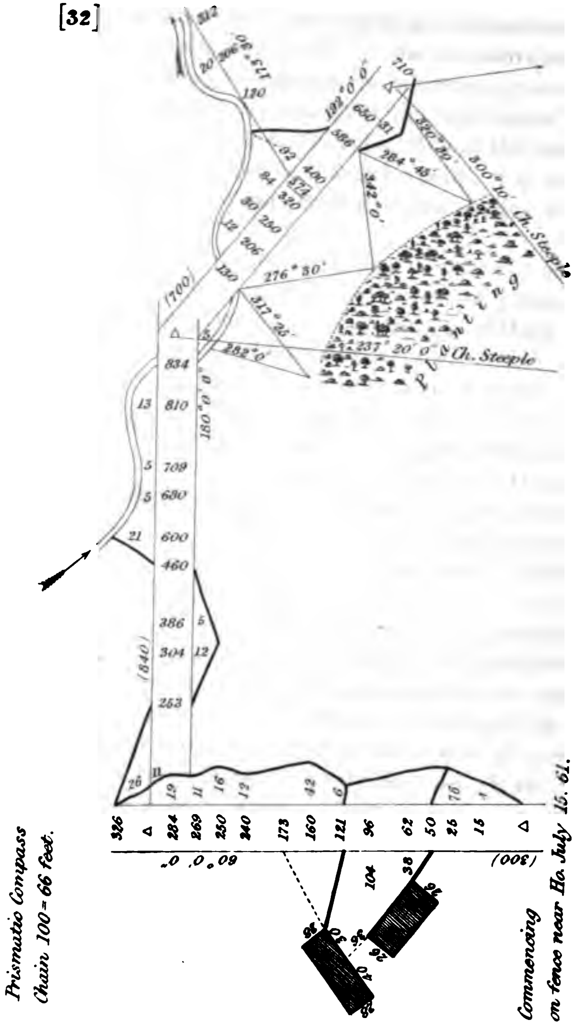
Let *a b c d e* (fig. 31) be the angular points of the traverse polygon, and let *m m′* be the circumscribed parallelogram for the computation of the area. If there be relative error in the measurements, the opposite sides of this parallelogram will not be truly equal. The lines or angles in error will not be distinguishable. Now if 45° be added to each of the angles, the parallelogram for computation will be as represented in the diagram by *g g′*. If the sides *b c*, *e d*, or an adjacent angle, be those relatively in error, the corresponding side of *g g′* will show this error more distinctly than will be shown by the sides of *m m′*. In this manner an indication of the point or part—the line or angle—in relative error will be obtained.

Fig. 31.



To localise errors sufficiently, so that the detail surveyor may remeasure the parts in error, a more perfect check is required. This may be effected by the measurement of angles from two or more angular points to an internal point of linear intersection. It has been already

[32]



H. Adlard Sc.

London: Longmans & Co.

shown, that when this check is really perfect for all parts of the polygon, the internal lines affording the check are the sides of the triangles of a triangulation of the enclosed district. In those cases the traverse becomes a triangulation.

The traverse polygon may be subdivided, by internal traverse lines, from any one known point to any other known point. The polygons so formed may be treated as independent traverse polygons.

It will be found convenient in traverse surveying to proceed with the sun, keeping the district to be embraced in the traverse on the right-hand side, as in the annexed traverse field-book (fig. 32). In the sketching and filling in of details, it may sometimes be an advantage to determine the position of a remote point by the magnetic angles of lines from known points in the traverse lines, so selected, that the angle made at the remote point may not be less than about 30°. The magnetic angles may be measured from the remote point to at least two known points in the traverse lines, or to other well-determined points. The measurements on a reference line for the required detail should commence at this remote point. If the detail be, generally, not well defined, such as the margin of marshes, bogs, brushwood, &c., the determination of the remote points may be sufficient for many purposes. Between the points so determined the features of detail may be sketched in with the hand.

CHAPTER IV.

PARTICULAR SURVEYS—SURVEY OF TOWNS.

THE importance of the accurate determination and delineation of the measurements for, and the position of, town detail (when laid down to a large scale), makes it desirable that the reference lines for this service be fully verified by the laying down of these lines on paper from the measurements on the field-book. For the more accurate determination and verification not only of the position and limits of the reference lines, but also for the independent determination of the chief points in the detail to be surveyed on those lines, it is desirable and important that the circumscribing base lines of the survey be sides of triangles of a trigonometrical survey, embracing the town and surrounding neighbourhood.

In 'poling' for the trigonometrical survey, regard should be had to the service and objects for which the survey is undertaken, so that the points selected for observation in measuring the angles shall be points easily accessible in the detail survey. If conspicuous permanent objects be not suitably located in the town, *flag-poles* should be erected in suitable and convenient positions. The trigonometrical survey should be complete, and the trig. and other observed points laid down from the computed distances to the proper scale for the map, before taking up the detail survey of the town. The principal reference lines of the town survey should be so selected as to

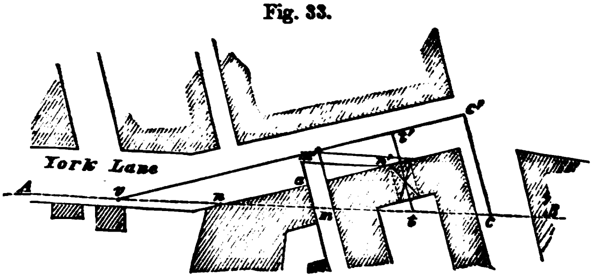
traverse streets the direction of which may facilitate the tracing of straight lines from reference points in the primary reference lines of the triangles for the detail survey of the suburbs on the one side to like points and lines on the other side. These town reference lines may be traced both ways, from an intermediate point, to intersect the primary lines in whatever point they may; or they may be traced from a given reference point in a primary line, through a particular intermediate point, and prolonged to intersect the lines on the other side of the town; or the town reference line may be traced from elevated remote points in the suburbs, to intersect the primary reference lines on both sides of the town, according to circumstances. Other town reference lines may be traced to reference on the town principal reference lines, or on these and the primary reference lines, or on one another, as may be found desirable for the purposes of the survey. Two reference lines should not have an intermediate common point of intersection, as the point of a reference line first intersected by another reference line should be the reference point and limit of the latter line. Two reference lines, however, may have a common reference point, and be in the same straight line. All town reference lines, of any considerable length, should be traced at numerous points with a theodolite, and the reference points accurately determined and properly marked for subsequent identification. The detail of those parts of the town which cannot be obtained on reference lines, connected directly with other reference lines, may, from necessity, be surveyed on traverse reference lines referred to the direct reference lines, or to the angular points of other traverse lines.

It may be seen that the town reference lines belong to

that class denominated 'blocking,' already referred to (see illust. 6). The measurements for detail should, however, be so made, that the ultimate point surveyed on one reference line may be also surveyed on the adjacent reference line.

In town surveys numerous inaccessible distances and obstacles to the direct continuous trace of reference lines will occur. These lines may be traced to and beyond the obstacles by adopting, in each case, some one of the modes already described for tracing obstructed lines. The theodolite should be used for laying off and tracing perpendiculars, angles, &c. It should be also used for the tracing of lines and laying off the angles for finding the inaccessible distances, except in certain cases where the chain may be preferred, as at street corners, such as *m m'* (fig. 33).

Fig. 33.



The mode of proceeding in finding inaccessible distances in town surveys may be, perhaps, better understood by the consideration of a practical case. In the illustration the detail line of the building at *a* enables the surveyor to prolong these lines accurately, in which case the theodolite need not be used. For the trace of the line at *c*, and the line *v c*, the theodolite should be placed over *v* in the reference line, and the angle *v*

($= c\,v\,c'$) laid off, and the line $v\,c'$ traced at c'. The line $v\,c'$ should be accurately measured with the chain. At c' a right angle, or the complement of the angle v ($= 90° - v$), should be laid off on the line side of $v\,c'$, and the line traced to intersect the reference line. From these data we have $c\,c' = \dfrac{v\,c' \times \sin v}{r}$, the distance of c' from the reference line at the point of intersection of $c\,c'$. The inaccessible distance $v\,c$ in the reference line will be found from the formula—

$$v\,c = c\,c' \cos v,$$

or $$v\,c = (v\,c'^{\,2} - c\,c'^{\,2})^{\frac{1}{2}}.$$

The reference line may be determined at an intermediate point, and traced in the rear of buildings as follows:— At a point t' in the line $v\,c'$ a right angle, or the complement of v, should be laid off on the line side of $v\,c'$. The line may be traced through a gateway, passage, &c., to the rear of the buildings and to the reference line. The distance $t'\,t$ may be computed by the proportion—$v\,c' : c\,c' :: v\,t' : t'\,t$. This distance being laid down determines t. At the points t and c the reference line should be laid off at the proper angle to $t\,t'$, $c'\,c$, and traced on the rear and front of the buildings. The distance $v\,t$ may be computed as above, and the measurements in the rear, &c., entered on the field-book, as in ordinary accessible detail.

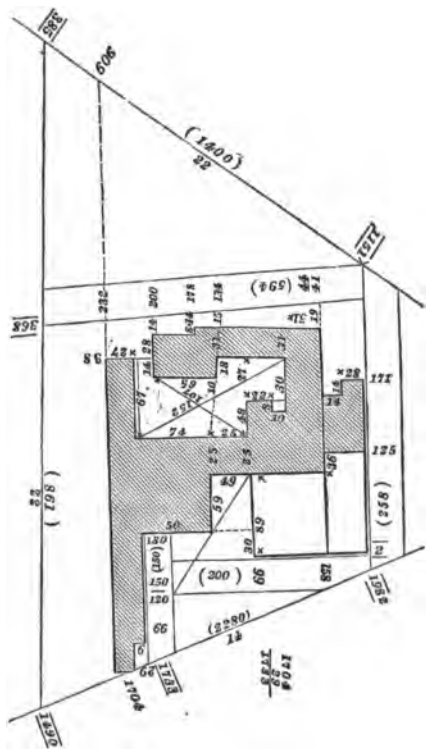
The line $v\,c'$ may be made a reference line, if desirable, by prolonging $c\,c'$ to reference on another reference line. In these determinations it is not absolutely necessary that $c\,c'$, $t'\,t$ should be perpendiculars to the reference line; when the circumstances permit it is desirable that such lines should be perpendiculars. In this, as in other surveys, offset triangles, with sufficient checks, are admissible if the detail cannot well be obtained on other reference lines.

The survey of detail on reference lines traced and measured, as described above, gives its position with much greater accuracy than when the detail is surveyed on a traverse of lines along the streets. The superiority of the survey on the secondary reference 'block' lines may be readily seen, as in this mode the front and rear of buildings and backward detail is alike directly obtained on the reference lines, the length and position of which are independently verified. The superiority may be also judged on a consideration of the accuracy with which the lines may be laid down on paper, and the direct verification obtained from the field-book, as well as the close proximity of the detail to the reference line for its survey.* In the traverse there is no absolute verifica-

* The town of Barajas de Madrid, in Spain, was surveyed by the author, in 1861, on secondary and primary reference lines. The survey was undertaken in this manner chiefly with a view to demonstrate to others the superiority of skilful over unskilful surveying, as a traverse survey was that approved by the field superintendent of detail surveyors. The town is in part skirted by olive (walled) gardens, and has an irregular circular outline. The streets are numerous, and it was found impracticable to trace a single unobstructed principal secondary reference line. The obstructions and inaccessible distances were comparatively numerous. The church steeple, near the middle of the town, was surveyed on a detail line, the survey of which was obstructed by ranges of buildings at three parts, the church being between two of these obstructions. The axis of the spire, as found by the detail survey, differed by 3 in., or rather less than half a link, from the position determined by the trigonometrical survey. The plan was laid down to a scale of 1 in 300, and subjected to a rigid examination. It passed through the ordeal without condemnation in any particular. [This is alluded to, to show the success attained, with inexperienced assistants, for a difficult town district, by the adoption of that system, or geometrical construction, in which the chain and scale measurements verify each other, and so the fidelity of the plan is assured before its detailed examination.]

tion of the position or length of a particular reference line practicable; and besides, the remoteness of the reference line from the rear detail, generally indirectly surveyed on it, is a serious difficulty, not only on the ground,

Fig. 34.



but also in the office—not only in the survey of the detail, but also in the laying down the measurements on paper.

In the survey of towns and buildings generally the perpendiculars should not exceed 20 to 40 links. From this it may be seen that in streets of considerable width it is according to good practice to run reference lines for the survey of detail on each side.

Each house with parallel sides should be surveyed by perpendiculars to three corners, and face measurements from these to the remaining angle or corner. The point of intersection, on reference lines, of the production of the lines of the face walls of chimneys, gables, faces of buildings, walls, &c., should be surveyed on different lines, and shown on the field-book, to verify the position given those faces by the detail measurements. The measurements of the internal details of buildings should be checked by diagonals, prolongations, &c., as it will be impracticable in some instances to obtain directly, on reference lines, measurements of such details. An inspection of fig. 34, illustrating a survey of buildings, will show the points requiring attention.

Surveys for Railway, Canal, Road, &c., Purposes.

Surveys for these, or like purposes, require to be conducted with much skill, and made with a high degree of accuracy. The chief features of the districts for survey in those services are great length, narrow width, and irregular outline. Having regard to the surface configuration, the special service in each particular case, the limitation of time, and the general financial interests of the projectors, the 'traverse survey,' with trigonometrical checks more or less perfect, may be in most cases advantageously adopted.

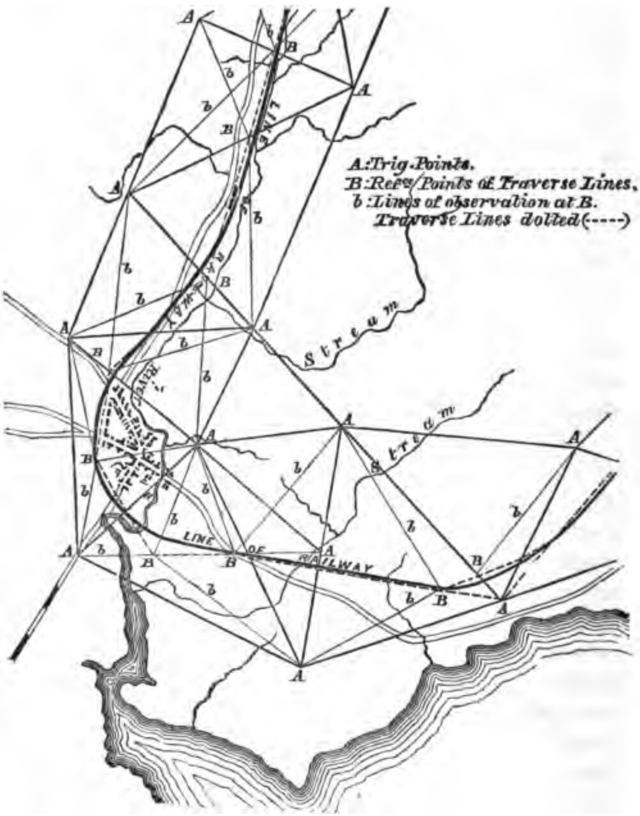
In these, as in other surveys, the surveyor should make himself acquainted with the position of the leading

features of the whole line of district, and the more particular features at certain points (which, if the district be extensive, should be obtained by a preliminary reconnoitering survey), before determining the particular locality and lateral extent of the district for detail survey. After the route of the detail survey operations is determined, the selected district should be 'poled,' so that the reference lines may traverse the triangles, of which the 'poled' shall be the angular points. The triangles so formed should be well conditioned. The sides of remote triangles should be accurately measured as *base lines*, and *base lines* for *verification*, so that the whole district may be simultaneously surveyed in sections, which, when applied to one another, shall make one continuous whole. The angles of the triangles should be measured with a suitable theodolite, and the sides computed before proceeding with the survey of detail.

The traverse may be conducted with the theodolite as already described in Chap. III. The points in which the traverse reference line intersects the side of a triangle should be determined by tracing the intersected side at the point of intersection. From these points the angular or trig. point (poles) should be observed, and the angles made by the side of the triangle with the traverse reference line, and the lines to the observed trig. points, measured. From these data the position of the traverse reference point in the side of the triangle, the distance from this point to the observed trig. points and the traverse reference line traversing the triangle, may be computed. The points and lines may be laid down from the computed distances. An inspection of the illustration (fig. 35) will suffice to show the reference and trig. points, and the lines and distances for computation,

and the projected line of the railway, for which the trigonometrical and traverse surveys may be combined.

Fig. 35.



With these additional measurements the *traverse* may be conducted as for an ordinary detail survey. Remote

detail may be taken up, if desirable, with the aid of the prismatic compass, as already described. The measurements, to the horizontal with the chain, of the traverse and other reference lines will be sufficiently checked, for most practical purposes, by the computed and plotted distances.* In some places the line of works will deviate from the traverse lines. For railway and some other works this deviation will be generally on a circular curve of lesser or greater radius. The deviation may, however, be along any other regular line.

TRACING CIRCULAR CURVES (FOR RAILWAY OR OTHER PURPOSES).

Circular curves of a known radius may be traced on the ground by any of the following and some other methods:—1. By the measurement of perpendiculars to certain chords produced. 2. By the measurement of perpendiculars to corresponding reference points in the chord. 3. By perpendiculars from reference points in the tangents. 4. By lines traced with the theodolite between reference points in intersecting tangents. 5. By the intersection of lines traced with the theodolite on particular angles from the tangent point — point of tangency—and the measurement of certain lines from adjacent known points in the curve. 6. And by the intersection of lines traced simultaneously on corresponding particular angles, with theodolites placed at the extremities of the chord points of tangency.

The authorship of some of these modes for tracing circular curves is disputed. It is believed Mr. Baker is

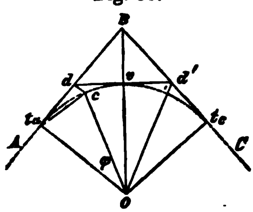
* In Ireland, and many parts of England and Scotland, the publication of the detail *six-inch* Ordnance Maps enables the engineer to lay down directly the line of works on these excellent maps.

prominently amongst the first and most original authors to publish modes for the purpose. The author disclaims any intention to investigate, in this place, the question of authorship where it may be disputed. It is proposed in this place to describe the best modes for tracing circular curves, without prejudice to the claims of the authors of these modes.

In every case the traverse reference lines, or tangents, on which the deviation commences should be traced and measured to their intersection or angular point, and the angle measured.

The marginal diagram (fig. 36) will assist the reader in understanding the following descriptions and investigations:—Let $A B$, $B C$ be the traverse reference lines intersecting in B; then will B be the measured angle; and let $t_a\, t_c$ be the points of tangency for a circular curve whose radius is $r\,(=o\,t_c)$. The distance of the points of tangency from B may be computed from the formula—

Fig. 36.



$$r \cot \tfrac{1}{2} B = B\, t_c\, (= B\, t_a). \quad \text{Also}$$
$$r \operatorname{cosec} \tfrac{1}{2} B - r = B\, v,$$
$$\text{or} \quad (\operatorname{cosec} \tfrac{1}{2} B - 1)\, r = B\, v,$$
$$\text{or} \quad (B\, t_c^2 + r^2)^{\frac{1}{2}} - r = B\, v.$$

This gives the curve point in the line bisecting the measured angle, which is the common secant to the tangents. Again we have

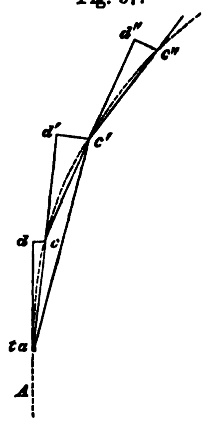
$$r \tan (45° - \tfrac{1}{4} B) = v\, d\; (= dt_a),$$
$$\text{or} \quad r \tan \tfrac{1}{2} (90° - \tfrac{1}{2} B) = dt_a.$$

The points t_a, t_c, d, d' and v may be determined by

measurements from the point of intersection B. The curve may be traced from t_a, v, or t_c, as follows:—

1. Let $t_a d$ (fig. 37) be part of the tangent (t_a being the point of tangency), $d c$ a perpendicular terminating in the curve, and ϕ the angle at the centre subtended by the chord $t_a c$. Then

Fig. 37.



$$t_a d = \frac{2 \sin \frac{1}{2}\phi \times \cos \frac{1}{2}\phi}{r}.\ \text{And}$$

$$d c = \frac{c t_a^2}{2r}.$$ This is the general formula for the perpendicular to the tangent.

The chord $t_a c$ should be traced through the forward determined point c and prolonged to d', making $c d'$ equal to $t_a d$. At d' a perpendicular $d' c'$ should be erected to terminate in the curve. To find the length of this perpendicular we have from the equiangular triangles $t_a d\ c$, $t_a d'\ c'$, $d'\ c' = \frac{t_a d' \times d c}{t_a d}$.* The lines $t_a c$, $c c'$ are equal; if, therefore, $c c'$ be produced to d'', making $c'\ d'' = c\ d'$, the perpendiculars $c''\ d''$, $c'\ d'$ terminating in the curve will be equal; and so on for other equal perpendiculars to the equal chords prolonged to equal distances. By tracing the chord lines, and laying down the equal distances and perpendiculars in succession, the curve will be traced at the points $c\ c'\ c''$, &c.

2. For tracing the curve through the extremities of

* When r is large with respect to $t_a c$, $2\, t_a d$ may be taken as equal to $t_a d'$, and therefore $d'\ c' = 2\, d\, c$.

perpendiculars to the chord, the length of the perpendicular and the position of its reference point on the chord may be found as follows :—

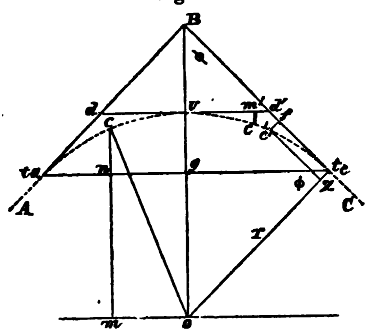
In the diagram (fig. 38) $t_a t_c$ is the chord, and g the point of bisection. Let the angle $g t_c o$ be represented by θ, and $t_c o$ by r; then

$o g = r \sin \theta$,

or $o g = (r^2 - t_c g^2)^{\frac{1}{2}}$;

and $2 r \cos \theta = t_a t_c$. This verifies the correct measurement of the chord. Again, let $o m$ be the diameter parallel to the chord. Then

Fig. 38.



$$(r^2 - n g^2)^{\frac{1}{2}} = m c. \quad \text{Hence } m c - m n = n c,$$

the required perpendicular from the reference point m.

In laying down or tracing, the curve $t_a t_c$ should be first measured and verified before marking the bisection at g. Measurements for the reference points m, &c., should have their zero in g, and be continued both ways to $t_a t_c$. The perpendiculars should be erected and traced with an angular instrument.

3. The curve may be traced through the extremities of perpendiculars to the tangent. These perpendiculars may be found as follows :—

Let $v m'$ (fig. 38) be the distance from the point of tangency v to the reference point m', and $m' c$ the perpendicular. Then we have from the general formula $\frac{v m'^2}{2 r} = m'c$.

It may be seen by an inspection of the diagram that

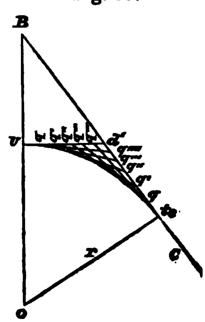
the perpendicular to the tangent terminating in the curve, at any particular distance from the point of tangency, is the complement of the perpendicular to the chord of four times the arc, at an equal distance from the bisection of the chord. It may be also seen that the perpendicular $f c'$ will be equal to the versine ($z t_c$) of the angle ϕ', of which $z c'$ shall be the sine, and that the distance from $t_c f$ to the reference point f will be equal to the sine ($c' z$) of the angle ϕ'. With a table of nat. sines and cosines, the distances for the reference points and their corresponding perpendiculars may be conveniently computed from the following formula by giving different values to ϕ:—

$$r \sin \phi \; (= z c') = t_c f.$$
$$r (1 - \cos \phi) \; (= z t_c) = f c'.$$

4. The distances from the points of tangency in intersecting tangents to the reference points of the lines tangents at certain intermediate points in the curve may be found as follows:—

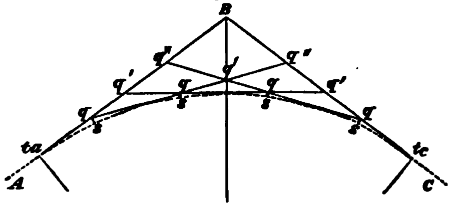
It has been already shown (p. 110) that the line $B v = (\text{cosec} \frac{1}{2} B - 1) r$. Also that $v d' = r \tan \frac{1}{2} (90° - \frac{1}{2} \text{B})$. Now let the arc $v\, t_c$ (fig. 39), (half the curve to be traced), be divided into m equal parts, and let ϕ represent the angle at the centre subtended by one of those parts; also let q be the distance in the tangent measured from the points v or t_c, so that when $\phi = o$, $q = o$, and when $\phi = t_c o v$, $q = t_c d'$ ($= v d'$).

Fig. 39.



In this we have the following relation:—

Fig. 40.



$$r \tan \tfrac{1}{2}\phi = q_1$$
$$r \tan \phi = q_2$$
$$\&c. = \&c.$$
$$r \tan \frac{m}{2}\phi = v\,d'.$$

By tracing straight lines through the points q_1 q_{m-1}, q_2 q_{m-2}, &c., on the tangents, the traced lines will touch the curve, and intersect the tangent next in order, at the distance $v\,q$ (fig. 40) from their respective points of tangency t. The traces of these lines determine directly the points of intersection q q q, &c., alluded to; and the point of bisection of the line $q\,q$ determines t, the trace of the curve at that part.

In this mode of tracing the circular curve, the line composed of the tangent both ways $(q\,t + t\,q)$ is constant, and so the measurements of and in these lines for the determination of the points $t\,t$, &c., will detect any line inaccurately traced or incorrectly determined. This ensures the correct trace of the curve on its first setting out.

If the distance $q v'$ $(=s) = (\operatorname{cosec} \tfrac{1}{2}\phi - 1)\, r$ be laid down in a line bisecting the angle made by the tangents at q, &c., other points will be accurately determined in the curve.*

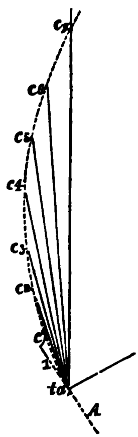
5. By combining angular and linear measurements, circular curves may be traced as follows:—

It has been already shown (p. 111) that the angles at t_a, the point of tangency, subtended by equal chords $t_a c$, $c\,c'$, &c., are equal. From this it may be seen that if the equal angles be laid off with a theodolite at t_a (fig. 41) and the lines traced, the curve points in the traced lines will be successively determined by the measurements of the successive equal chords, to terminate in the point of intersection of the chord and traced line.

* The author believes this new method to be of great practical value.

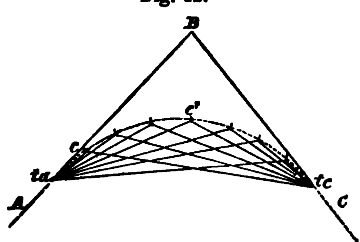
If ϕ be the angle at the centre subtended by $t_a c$, then the angle to be laid off from the tangent at t_a will be $m \times \frac{1}{2}\phi$, m being a whole number. The curve cannot be conveniently traced when $t_a c$ exceeds one chain in length. In practice it is desirable to determine the successive curve points by a whole chain measurement for the chord simultaneously with the laying off and trace of the proper angle and line. If the radius of the curve be considerable, so that one half the angle subtending a chord of one chain shall be so small as to render the trace of the line in the field unsatisfactory, a larger theodolite than is required for the other purposes of the survey, and great refinement in measuring the chord, would be necessary to meet the requirements for the correct trace of the curve.

Fig. 41.



6. The curve may be traced through the points of intersection of lines containing equal angles from the extremities of the chord. In the diagram (fig. 42) the corresponding lines drawn from the extremities of the chord t_a t_c make equal angles at $c\ c'$ in the same segment. Hence $c\ t_a t_c + c\ t_c t_a = c' t_a t_c + c' t_c t_a$. Rejecting the angles common to both sides of this equation, viz., $c'\, t_a\, t_c$ and $c\ t_c\ t_a$, the remainder $c'\ t_a\ c = c\ t_c\ c'$. It may be easily seen that this relation obtains for all positions of c and c', and that as the angle at

Fig. 42.

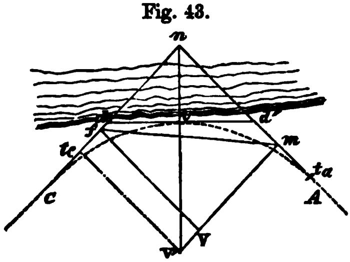


t_a increases, the corresponding angle at t_c will decrease to an equal extent. The curve may be traced through the points of intersection of the corresponding lines traced with theodolites placed at t_a and t_c, at which points the proper angles should be laid off. The increase or diminution in the angles may be by a constant or variable angle, which should have, as already stated, different signs at the tangent points or extremities of the chord. If the difference between the immediate successive angles be a constant, the distance between the determined points should be found to be also a constant.

In tracing the line of works on the banks of wide rivers, the margins of lakes, and the skirts of dense underwood, &c., it will sometimes occur that the point of intersection of the tangents or traverse lines will be inaccessible, so that the necessary measurements made from and at this point cannot be directly obtained. In these or like cases, which are similar to the class of cases for finding inaccessible distances to an inaccessible point, the measure of the angle contained by the tangents, and the determination of the tangent points, may be ascertained as follows:—

Let $A\,n$, $C\,n$ (fig. 43) be the tangents, and n the inaccessible point. Let m be a convenient intermediate point in $A\,n$, and f a like point in $C\,n$, f being visible from m. The angles $C\,f\,m$, $A\,m\,f$ should be measured. These angles are each equal to the sum of the angle n and the supplement of the other angle. Or,

Fig. 43.



$$A\,m\,f + C\,f\,m = 2n + n\,m\,f + n\,f\,m.$$

By deducting the latter or known angles from both members of this equation, we have—

$$(A\,m\,f - n\,m\,f) + (C\,f\,m - n\,f\,m) = 2n. \text{ And}$$
$$\tfrac{1}{2}((A\,m\,f - n\,m\,f) + (C\,f\,m - n\,f\,m)) = n,$$

the angle contained by the tangents. At f and m the angles $C\,f\,V$, $A\,m\,V$ should be laid off, each equal to n, and the lines traced to intersect in V, an accessible point. In the parallelogram $V\,f\,n\,m$, the angles V and n are equal; and the lines $m\,V$, $f\,V$ are equal to $f\,n$, $m\,n$ respectively. The lines $m\,V$, $f\,V$ should be measured, and the shorter prolonged and made equal to the longer; or the length of the shorter may be taken up in the longer line, so that the sides of the parallelogram, having the latter determined points, and n opposite angular points, shall be equal. In the diagram $m\,V$ is prolonged to V', so that $m\,V' = f\,V$. Then $m\,V' = m\,n$, and the angles $f\,n\,V' = m\,n\,V'$. The line $n\,V'$ bisects the angle n and the curve between the tangent points. The radius of the curve and the angle n being known, the distance from n to t_a, t_c—the points of tangency—may be computed by the formula already given (p. 110). If from $n\,t_a$, $n\,t_c$ the known equal distances $n\,m$, $n\,f$ be separately deducted, the remainders, $m\,t_a$, $f\,t_c$, may be laid down by measurement, and the points of tangency so determined. The diagonal $V'\,n$ may be computed, and also $n\,v$, the former from the formula $n\,V' = m\,V'\,2\cos\tfrac{1}{2}n$, and the latter from the formula p. 110. From these $V'\,v$ may be readily found. At V' the angle $m\,V'\,v\ (= \tfrac{1}{2}n)$ should be laid off and the line traced to v, which should be determined by measurement from V'. At v, $d'\,d$ a perpendicular to $V'\,v$, and tangent to the curve, should be traced, and the points $d\,d'$ determined. It should then be found that $t_a\,d$ shall be equal to $v\,d$—half of

$d' d$. This will verify the determination of the points $d' v d$.

The point v may be also determined by laying off from the tangents at t_a, t_c angles each equal to $\frac{1}{4}(180-n)$, and tracing the lines to intersect in v. From this point the tangents $v d$ may be traced by laying off the angle $d v t_a$ equal to that laid off at t_a. The point d may be otherwise determined by measurement from m, as an ordinary intermediate point in the tangent, if d be an accessible point. The line $t_a v$ is not shown on the diagram.

Remarks on the Tracing of Circular Curves.

It may be advisable, before passing from the tracing of circular curves, to consider generally the practical features of the problem. It is sufficiently ascertained that lines may be traced with a theodolite, in correct adjustment, for very considerable distances—even for several miles—with a high degree of accuracy. It is equally well ascertained that the several numerous operations in connection with the measurement of distances with the chain, and overcoming obstructions in the field, cannot be practically carried out free from error, even by the most painstaking surveyors. These errors will be of uncertain amount, as they depend on the distance to be measured and the difficulties to be overcome. Comparing the practical results obtained, it is found that the errors occurring in the determination of points by measurements with the chain greatly exceed those which occur when the points are determined by the intersection of lines traced with a theodolite. This superiority by trace is reasonable, as the lines are traced without regard to surface irregularities at intermediate

points, which irregularities are a serious cause of error in making horizontal measurements with the chain.

It may be readily seen, by a little consideration, that the measurement of short distances in traced tangents, for the point of tangency at any part of the curve, determines the curve at that part more accurately than when the determining measurement is made in a line making an angle with the tangent. In the former case the curve and tangent coincide, without practical or appreciable error, for a very appreciable distance from the point of tangency, and so the unavoidable error in the measurement with the chain does not erroneously determine the trace of the curve, the accuracy of which depends mainly on the trace of the tangent. In the latter case, any error in the measurement of the distance, assuming the line to be correctly traced, will introduce a corresponding inaccuracy in the trace of the curve so determined. It may be seen that this inaccuracy in the trace of the curve will be greatest when the measured line is perpendicular to the tangent at the point; that it diminishes with the angle made with the tangent; and that it vanishes when the angle vanishes, which is the former case.

It may be observed that the curve will be more accurately traced when the determination of the several points in it is made independently, and in a manner to afford a verification, than when the determination of one point is made by measurement from or on lines traced through other like determined adjacent points.

Division of Lands.

Surveys for the 'division of lands,' and surveys for a map, &c., already described, are somewhat differently conducted. In the former some of the measurements

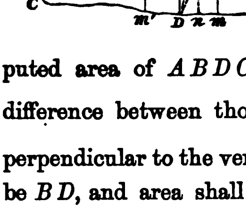
are determined by computations on a given area or quantity, in the latter the measurements are determined by certain lines independently of quantity or content.

In the 'division of land' the cases of most frequent occurrence will have all the sides of a more or less regular polygon given, except one or two indeterminate and an undefined adjacent side. In many cases the position of the required side and its abuttal on existing detail will be undefined.

It is not proposed to give an example of each particular possible case which may arise (this would be of little practical value), but it is proposed to make clear the main points to be attended to, leaving it to the surveyor to suit the application of what is given below to the circumstances in each case.

In the diagram (fig. 44) the side AC is given in position and magnitude, and the sides AB, CD in position only, being indeterminate towards D and B. The line BD should be selected, so that the area $ABDC$ may be approximately equal to the given area. This approximate parcel should be surveyed and its area found. If A represent the given area, and a the computed area of $ABDC$, then $A-a$ will represent the difference between those areas, and $2\left(\frac{A-a}{BD}\right)=p$, the perpendicular to the vertex of a triangle, whose base shall be BD, and area shall be equal to $\overline{A-a}$. In geometry it is shown that if through the remote extremity of the perpendicular, p, a line be traced parallel to BD, all triangles having BD for their base and their vertex in the

Fig. 44.

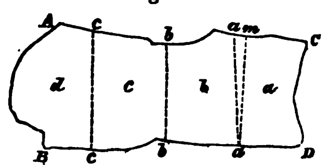


parallel line will be equal to one another, and to the difference of areas $A-a$. In the diagram $m\,m$ will be the parallel line when $A>a$, and $m'\,m'$ the like line when $A<a$. If the perpendicular p be bisected, and a parallel line $n\,n$ traced through the point of bisection, it will be the required boundary, if the lines on which it abuts be parallel. If it be desirable to incline the boundary line so as to make it abut on the adjacent boundary lines at B or D, the parallel line $n\,n$ should be bisected. Any line traced through the point of bisection to intersect the adjacent parallel, or approximately parallel, lines may be selected as the required boundary line. A little consideration will make this clear.

If it be required to subdivide, proportionally, a parallelogram—figure with parallel sides—the division may be easily made by dividing a side in the given proportion, and tracing lines through the points of division parallel to the adjacent sides. The areas of the parallelograms so formed are proportional to their bases or the sides in the same line.

If, however, the outline of the parcel or district be irregular, as in many cases it will be, the division into proportional parts must be determined differently. The division may be made as follows:—The area of the whole parcel or district should be found from a survey made for the purpose or otherwise. Let $A\,B\,C\,D$ (fig. 45) be the parcel for proportional division, and let $a\;b\;c\;d$ be the parts into which it is required to divide it. The proportion and number of the parts will depend

Fig. 45.



on circumstances. For the number of parts given above, let the proportion be as follows:—

$$\begin{aligned} a &= 2 \quad \text{parts} \\ b &= 3 \quad \text{"} \\ c &= 2{\cdot}5 \quad \text{"} \\ d &= \underline{1{\cdot}6} \quad \text{"} \end{aligned}$$

Total number of parts $9{\cdot}1$. These parts represent the area of the parcel $ABCD$. Let this area be represented by A, then the area of the subdivisions will be

$$a = \frac{2 \times A}{9{\cdot}1}, \quad b = \frac{3 \times A}{9{\cdot}1}, \quad c = \frac{2{\cdot}5\,A}{9{\cdot}1}, \quad \text{and } d = \frac{1{\cdot}6\,A}{9{\cdot}1}.$$

The sum of the computed areas should be equal to A.

To determine the division a: Let A' be the area of an approximate parcel $amCD$, then $\frac{2 \times (A - A')}{9.1}$ will be the difference of areas to be laid off on the line am. From these data the boundary line $a\,a$ should be laid down as already directed.

The parcels $b\,b\,a\,a$, $c\,c\,b\,b$ should be determined in succession in like manner. It should be then found that the area of the remainder $AB\,c\,c$ shall be equal to $\frac{1{\cdot}6\,A}{9{\cdot}1}$. In this manner the proportional areas of $a\;b\;c\;d$ should be determined and verified.

This is the general case, of which there may be many particular modifications. For instance, it may be required that an extremity of the dividing lines shall meet. This only requires that the lines selected for the approximate areas should be run from the same point in the boundary of the parcel. Other particular cases may be based, according to circumstances, on the general case just given.

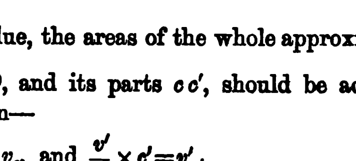
PROPORTIONAL DIVISION OF LAND, ACCORDING TO VALUE.

For this purpose the value of land may be taken as comprised of two elements, namely, area and some quality or special convenience. The value may be therefore expressed as *area* × *quality*. Now, taking the area as constant, the value will be proportional to quality; and taking the quality as unvariable, the value will be proportional to the area. Let A represent the area, and q the quality. Then $A \times q = v$ the value. And

$A : A' :: q' : q$ (for parcels of equal value). This is an inverse proportion.

Let it be required to divide the parcel $A\,B\,C\,D$ according to value (fig. 46)—say, into the same number of parts and in the same proportion as given above for area only. In this parcel let there be two qualities, $a\,a'$, and let $s\,s$ be the quality dividing line. The area of the divisions of quality should be obtained from a survey. Let a and a' represent the area of these divisions, and let the values per unit of area be v and v' respectively. Also, let A represent the total area, or area of the parcel, and V its value.

Fig. 46.



Now $\frac{v}{a}$ = quality of a, and $\frac{v'}{a'}$ = quality of a'.

Taking the first proportional division P_1, its value will be $\frac{2\,V}{9{\cdot}1}$. To find this value, the areas of the whole approximate parcel $m'\,m\,C\,D$, and its parts $c\,c'$, should be accurately found. Then—

$$\frac{v}{a} \times c = v_c, \text{ and } \frac{v'}{a'} \times c' = v'_{c'}.$$

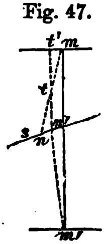
From this we have—

$$v_c + v'_{c'} = \frac{2\,V}{9\cdot1} \pm d.$$

In this formula d is the difference of value between the value of P_1 and that of the approximate part $m'\,m\,C\,D$. The line $m\,m'$ should be divided in the inverse proportion of the qualities, viz., $\frac{v'}{a} : m\,n :: \frac{v}{a} : m'\,n$. From n a line $n\,n'$ should be traced towards the value dividing line $s\,s$. The perpendicular $\frac{2\,d}{m\,m'}$ should be laid down from $m\,m'$, and through its outer extremity a line should be traced parallel to the line selected for the approximate parcel. Let n' be the point in which the parallel line intersects $s\,s$. Lines traced from n' to m' and m will be the required boundary lines for the parcel P_1 of the value $\frac{2\,\mathrm{V}}{9\cdot1}$.

If the difference of the qualities be considerable, or the point n be not near to the line s, it will be preferable to divide d in the inverse proportion of the parts of $m\,m'$ on the opposite sides of s (fig. 47). For this division of d, we have the corresponding areas on the opposite sides of s, $\frac{d_1}{q} = c_1$, and $\frac{d_2}{q'} = c_1'$. Hence $\frac{c_1}{m\,s}$, $\frac{c_1'}{m'\,s}$ will be the required perpendiculars. The boundary lines laid down to these perpendiculars, so as to have a common line in s, will be $m\,n\,n'\,m'$. This may happen to be an irregular and undesirable boundary line if n and n' do not coincide. To make it regular, the line $m\,n$ should be bisected in t, and $n'\,t$ traced to t', as already explained. Then $m'\,n'\,t'$

Fig. 47.



will be the required boundary for the given value of P_1. The other proportional parcels may be laid off in like manner.

Reconnoitring Surveys.

Reconnoitring surveys are required for many engineering and other purposes, for which it may be desirable to investigate the natural features of an extensive district or range of country, as for some colonial purposes. The positions determined in reconnoitring surveys are only approximate; and from the manner of proceeding, which shall be presently described, the mapping of the district may be completed in a comparatively short space of time.

In schemes for reaching and exploring the interior districts of an unfrequented country, or for determining the range, features, &c., of a country for settlement, and for many other colonial purposes, a reconnoitring survey is indispensable. The features in such service should embrace the physical structure or surface configuration, the subsurface or geological structure, the vegetable kingdom, animal life, and the distribution of rivers, streams, &c., &c. The fulness of the required detail will depend on the objects for which the service shall be undertaken, the limitation in respect of the scope of the survey. For some general surveys the watershed lines and their corresponding rivers and tributaries, stiff or shelving ranges, low grounds, lines of particular vegetation, lakes, rivers, springs, practicable roads, divides, passes, and general geological structure when exhibited in the sketching by etching and colours may suffice for map details.

Rapidity of execution will be a primary consideration in the field. The instruments for making the determinations should be very portable, such as a prismatic

compass or box-sextant for measuring angles, a well-appointed mountain barometer for estimating altitudes, field telescopes, &c. The instruments for the map roller and sketch-case should be scales, station pointer protractors, small parallel ruler, &c.; a supply of tracing paper, transfer paper, superior tissue paper, and a supply of light, strong, well-finished drawing-paper, together with hard and soft indelible pencils, tissue paper note-books, with transfer paper and ass-skin sheet, penknives, &c.

In this, as in other surveys, it is desirable to obtain a verification of the position of points determined independently. For this and other purposes, as well as to facilitate a close approximation to accuracy, rapidity of execution, and a diminution of the necessary labour, it is recommended that the survey of any considerable extent of country be made by, at least, two survey parties at the same time.

After the scale of the map is determined on, the first consideration in the field is the determination of the length of a suitable *base line.* This cannot be conveniently obtained by measurement with the chain, but it may be conveniently obtained in remote country with sufficient accuracy by attaching on the saddle post a reel of cotton thread, say ten miles in length. The free end of this thread should be fixed at one end of the *base line*, at which both surveyors should measure angles to well-defined feature points. The surveyor measuring the *base line* should ride in the selected straight line until he arrives at the other end of the base line, when the whole of the thread shall be paid out. At this point angles should be also measured to the same well-defined feature points, and to other like points. The points so determined by both surveyors should be laid down on their maps. This is the extent of the surveyors'

labours in common—the one, suitably provided with pack horses, and attended by assistants (and guides if practicable), taking to the high grounds; and the other, alike appointed, provided, and attended, taking to the low lying part of the district.

The surveyor on the plain should determine generally the course of rivers and their points of junction, the geological features, &c. The surveyor on the high ground will not require the use of canoes or bags for compressed air, supplied to the party on the plain. He will require other means of transport, to enable him to convey the mountain barometer for estimating the altitudes of passes, fords, divides, &c.

It will mainly depend on the arrangement of the features, the points at which the low-ground and high-ground survey parties may meet in the passage of the latter party from one side to the other of the valley of a stiff district. The surveyors should provide for meeting at these points, to obtain information, exchange sketches and notes, and arrange for the next point of meeting.

The well-defined feature points selected in the progress of the survey as the chief angular points of the geometrical construction, should be determined by the intersection of at least three lines, which should make angles not less than 30°. The distance from one such point to another should not exceed ten to twelve miles if practicable. From this it will be seen that the construction lines for the map will be the sides of triangles, through which a traverse of the surveyor's line of travel will run.

At about the distance of sixty to one hundred miles from a *base line*, another *base line* should be measured in like manner, to verify results, and give an additional element of accuracy to the map. All notes not transfer-

able at once to the map should be entered in duplicate for exchange with the companion surveyor at the next point of meeting, or the next point of intersection of the surveyors' lines of travel.

Colonial Surveys.

The governor, officer, or minister of lands should be in possession of a good skeleton map of the colony, previously obtained by reconnoitring survey parties, and the notes and reports taken and furnished by the survey party, and other authentic information, before proceeding to the consideration of the extension of the *Intake of Land* for *Colonial Settlement.* The officer, after sufficiently informing himself of the general features of the locality, should arrange the general features of the disposition of the new allotments, and draw up instructions to the topographical engineer in chief. These instructions should be accompanied with such information, relating to the locality for survey, as may be available.

The instructions, so far as topography shall be concerned, should require :—

I. That all measurements made in the field shall be redeterminable, at any subsequent time, by a surveyor of average skill and ability, from the field-book and permanent field marks.

II. That important reference points be clearly and permanently marked, and fully described by sketches and measurements of lines or angles.

III. That the principal permanently marked local points be observable from at least three trig. points, to be particularly described.

IV. That in each lot, or plot, there shall be at least three such reference points in suitable localities,

and that all of these points, not trig. points, shall be referred to trig. points particularly described by angles, &c.

V. That all measurements with the chain be made to the horizontal.

VI. And that daily meteorological observations be made, and a journal kept during the service, which should contain the daily reading of barometer, thermometer, rain-gauge, and wind-motor.

Farther, the topographical engineer-in-chief should be required to furnish the following documents as a return to these instructions:—

a. A section of the course of rivers surveyed, referred to the same datum, showing banks, water, and bottom traces; and showing also, by suitable figured indications, the velocity of the current and the computed volume or flow.

b. A longitudinal section of the lands reserved for public ways.

c. A geographical index map, lightly hill shaded, showing the outlines of district surveyed for allotment, public ways, reservations, water-courses, and trig. points—to a scale (say) 1 in 10,000.

d. A parcel, or reference allotment, map, showing by colours reservations for public ways and character of vegetation; also rivers (referenced for sections), and the surface features, if bold or remarkable—to a scale (say) 1 in 4,000.

e. Two copies of a map of each parcel or allotment plot, showing fully the details, and the lines and reference measurements for laying down the map and refinding the reference points—to a scale of (say) 1 in 2,000.

f. Two perfect copies and the original field-books of the survey containing the entries of the measurements from which the maps were laid down, and the books of computation for ascertaining the lengths of lines, areas of parcels, &c.

g. A plot of principal reference lines (two copies) distinguishing triangles, and giving the angular measurements for the permanently marked reference and trig. points; also a plot of the reference lines of each triangle (two copies). These plots should be laid down to a convenient scale.

h. A detailed report of the service, men and means, meteorological observations, depths and general qualities of the soil, geological structure, peculiarities, &c., &c.

The topographical engineer charged with making the return to the instructions, should first direct his attention to the selection of trig. points for a primary triangulation of the district. The selection of these points will allow the staff for the survey to be told off to the preliminary labours of the service.

At each primary trig. point a large flat stone, or levelled rock, should be prepared and permanently erected, about two to three feet below the surface of the ground, and into the middle of this stone a brass or copper batt bolt should be securely let, on which to mark the trig. point. Simultaneously with this erection, that of an elevated conical hut, having the trig. point in its axis, should be proceeded with, to serve as a conspicuous object for observation, and a place of refuge and security for the men and instruments. The hut should be supported on poles, with its floor about twelve feet from the ground. The sides of the hut should be double sheeted with boards for the first ten feet; above that, a single sheeting of boards

may suffice. The height of the apex of the cone above the ground will depend on local circumstances. The top or apex should be furnished with a suitable contrivance for exhibiting a magnesium, lime, or other light when desirable. The primary trig. point objects should be completed with the utmost despatch, and a reconnoitring survey made for the sites of the trig. points for the secondary triangulation, and also for the selection of a suitable *base line*, or 'combined measured bases.' Simultaneous with this survey of the surface configuration, the erection of the trig. point objects for the secondary triangulation, and the measurement of the angles of the primary and secondary triangulations (which may be taken up by separate parties), a contour survey* should be made of the vicinity of each primary trig. point, to a distance of half a mile, or to a decrease in elevation of about two hundred feet. This will give a full description of the position of the primary trig. point. The contour planes may be about three feet apart.

In the lines of the primary and other triangles, and distant from a primary trig. point as far as may be within their contour district, a suitable permanent object should be erected below the surface of the ground, and a brass bolt, similar to those for marking the trig. points, let into it. On the top of this bolt, made flush with the levelled surface of the permanent object, the side of the triangle should be accurately traced, and finely but deeply marked. To each line so marked the initial character of its trig. points should be stamped on the bolts, and also the corresponding corrected reading for an average arc. These points, after being so marked, should be carefully covered over, to preserve them from

* See 'Contour Surveying.'

injury. The line points preserve on the field the information for refinding, or restoring, the primary points from one another; and also furnish data for restoring, or refinding, the secondary trig. points from the field-book of angles.

In this, as in all trigonometrical surveys, the site of the *base line*, or *combined measured bases*, should be as near as may be in the vicinity of the centre of gravity of the district; except for some special convenience, it may be deemed advisable to select it near the outer part. The survey party may take up the measurement of the *base line* immediately on its being selected, so that its correct length may be ascertained in time to allow the computation of the sides of the triangles to be taken up on the completion of the measurement of the angles for connecting the extremities of the base line with the other trig. points. On the length of the lines of the secondary triangulation being computed, and a plot of the lines laid down on paper, the detail surveyors may be told off for progress in the field.

It will depend on the 'instructions' to the topographical engineer-in-chief—the character and extent of the surface obstructions—the particular class of survey to be adopted for the parcel details. In heavy wooded country, a traverse survey will generally be preferred; whilst in close wood a trig. survey of detail points and intermediate traverse will give the required data with sufficient accuracy and in a short period of time. In open country, and wherever practicable, the detail triangulation is preferable to the traverse, even when referenced to the trig. points of a trigonometrical survey. In these surveys the angles should be measured with the theodolite and a large prismatic compass (mounted), so as to obtain an indication of the local magnetic influence,

if any. This data will be available for laying down the lines and measurements with the aid of either angular instrument. The meridian line should be determined at some of the primary trig. points, and by angles connected with the other trig. points. In the traverse, the meridian line and zero diameter of the theodolite should coincide.

The point of intersection of the sides of triangles with traverse detail reference lines should be permanently marked, and the contained angles measured at these points. In addition to the mark at the point of crossing, a second mark should be made in the trig. line, at a measured distance from the mark at the point of intersection, to enable a surveyor to take up the retrace of the lines at the part. The reference point of a split line should be marked in like manner.

The latitude and longitude of the several trig. points should be computed from the origin of the axis of co-ordinates, coinciding with one of the primary trig. points. Over the origin of co-ordinates, a suitable permanent supersurface object should be erected. All the other trig. points should be marked above the surface by piles of earth, stone, &c., to assist in the subsequent ready identification of the trig. point.

At least one copy of each map, section, diagram, or document should be made in the outlying district, and also verified copies of each original document, note-books, &c. These documents should be preserved with care; copy extracts only should be furnished to the surveyors in the field. All secondary documents (or traces of them), such as maps and sections, &c., should be examined on the ground, and corrections investigated and made before the final drawing of the map shall be made.

The minister or officer of lands, on receiving the

return of the engineer-in-chief, together with an index register, should have a proper register made of all the documents returned to him. These documents should be secured against accident by depositing a certified copy of each in a suitable place or places for public purposes.

The conveyance of an allotment or parcel to a purchaser should recite and include a copy of the maps, measurements, and other parcel documents, certified copies of which should pass to the purchaser, as the description of the grounds. By this means the purchaser may be put into formal possession without going on the grounds, and restrained in case of encroachment on the surrounding parcels or public reservations. The documents so to pass would furnish, ever after, a complete description and the means of identification of the parcel or plot conveyed.

Intake of Land for Railways, etc.

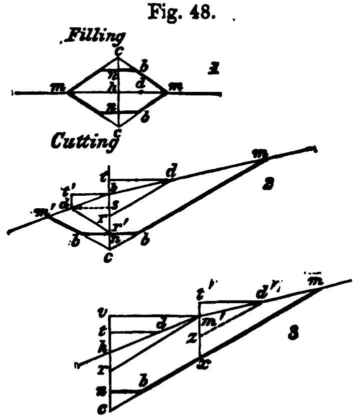
After determining the 'centre line' levels, and extent of works, it is necessary to make a survey of the 'intake of land' required for the purpose of the railway, &c., &c. In the field operations for the 'working section,' the centre line should be pegged at measured points, usually about sixty-six to one hundred feet apart. These peg points are taken up in the levelling of the centre line, and are shown as level points on the working section; their reduced levels should be inserted in the proper column on the field-book of levels. The section should show the extent of works, or the grade lines for the earthworks, and also the reduced level for the grade line, at each peg point. The inclination of the side slopes should be also shown. This information should be in the hands of the surveyor making the survey for *intake*, and also a working

plan showing the pegged centre line, intersected detail, &c. The surveyor should take levels, and other measurements, in a transverse line at each *peg*, to determine the surface inclination and limits of intake.

These measurements may be made, and the half-widths from centre line computed, and laid down as follows :—

Let $m\ m'$ (fig. 48) represent the surface line on cross section (determined by the surveyor in the field); h the centre line at level point; $h\ n$ the depth of cutting, or height of filling, as the case may be; $n\ b$ half the width at grade level $m\ b$ side slope, produced to intersect the vertical of h in c.

Fig. 48.



The level of a point d in the transverse line $h\ d$ should be found, and the distance of the point from h measured. Now, if there be no difference of level between the points h and d, as in (1), we have $\frac{h\,c}{n\,c} \times n\,b = hm$, the surface half-width to side slope. If, however, there be a difference of level between the points h and d, such as $t\,h$ (2), the surface half-widths will not be equal. These may be found as follows :—

The surface distance hd, and the corresponding vertical height th, are given. Then $dt = (dh^2 - ht^2)^{\frac{1}{2}}$. By taking $d\ t$ as a half-width on the level, the corresponding depth $t\ r$ to intersection of side slope may be found from the

formula $\frac{n\,c}{n\,b} \times d\,t = t\,r$. From this $h\,r$ may be readily found; and from the similar triangles $h\,r\,d$, $h\,c\,m$, the surface distance $h\,m$ may be found from the proportion $r\,h : h\,d :: c\,h : h\,m$. In like manner, the surface half-width $h\,m'$ may be found.

If the inclination of the surface line of half-width be not the same as in (3), the side right-angled triangle $h\,t\,d$ should be found as above, and the surface line $h\,m'$ measured. Then $h\,v = \frac{h\,t \times h\,m'}{h\,d}$, and $v\,m' = \frac{t\,d \times h\,m}{h\,d}$. From these $v\,r$ may be computed as above, and $r\,c$ $(= m'\,n)$ readily found. Now m' may be treated as a centre line point for a depth $m'\,x$. The triangle $m'\,t'\,d'$ and the depth $t'\,z$ should be found in the same manner as the triangle $h\,t\,d$ and depth $v\,r$ were found. Then $m'\,m$ should be ascertained for the depth $m'\,x$, as explained in the second case. The surface half-width $(h\,m' + m'\,m)$ should be ascertained in this manner. The model scale (illust. 82) may be, in many cases, conveniently used for this purpose.

To the surface half-widths, ascertained as above explained, the widths for fences, drains, &c., should be added to find the extreme half-width, terminating in the boundary line of the intake. The results thus arrived at do not, however, hold good on sidelong ground when the surface inclination varies on the consecutive cross-sections, at peg points on the centre line. To bring out strictly correct results the transverse half-width section line should be selected according to the surface configuration, and not arbitrarily at peg points. It will appear, on a little consideration, that on sidelong ground the half-widths for opposite sides of the centre line should not be referenced to the same point in that line. The trans-

verse lines for the half-widths will be most conveniently measured in vertical planes, perpendiculars to the vertical plane of the centre line at the reference point. In these determinations of the half-widths it should be found that the straight line joining the outer extremities of each consecutive pair of half-widths laid down shall be a straight line in the surface of the ground.

A much more exact mode of laying down the boundaries of intakes for railway or like purposes will be explained in connection with 'intakes,' Chap. VII.

The part of the areas of the several parcels included in the intake may be computed separately from surveys, and the results verified by taking the sum of the products of the successive mean half-widths, multiplied into their respective peg distances, or otherwise, by the Simpsonian rule.

SURVEYING SUBMERGED DISTRICTS.

When it may be desirable, for engineering or other purposes, to ascertain the surface configuration of a submerged district, such as a river bed, the bottom or bed of a lake, estuary, bay, &c., a hydrographical survey should be made in such manner as will furnish data for the representation of the features on a map. A preliminary or base map of the water margins and adjacent dry ground, if one already made be not procurable, should be made from a proper survey of the dry ground district. The prominent features and well defined surface objects should be clearly shown on this map. As the surface of the bottom is not generally visible, its configuration must be ascertained by referring numerous points in it to a known plane, such as the surface of still water. If the water surface be inclined, or not steady at the same level, as in a river with an appreciable current, or a tideway where

the surface fluctuates rapidly through a considerable range at the same point, the inclination of the river water surface should be obtained by levelling, and the fluctuations due to tide flow by observations on a tidal gauge erected, for the purpose, at suitable places.

The depth of the water from surface to bottom is generally obtained with a wire, or cord, weighted at the lower end. This is usually called a 'sounding lead and line,' and the point a 'sounded point.' The depth is called the 'sounding' of the point.

The bottom of the 'lead' is generally made concave, and filled with some stiff, sticky, and greasy substance, for taking up loose matter at the bottom, such as sand, &c. If the depth be less than 100 feet it is preferable to use a wire chain, or steel tape line, as a hempen cord varies its length with the degree of saturation.

The horizontal position of the sounded point may be determined from land or on water. If from land, it will be found convenient to observe simultaneously at two distant known points on land the surveyor's signal flag. The contained angle at the sounded point should not be less than 30°. The position may, however, be more conveniently determined by measuring with the sextant at the sounded point the angles to three known land objects, or it may be determined, though less accurately, by measuring the magnetic angles of the lines to two or more well defined land objects intersecting in the sounded point.

If the position of two remote sounded points be found by any of the methods just alluded to, the position of intermediate points in the straight line may be found by computation on the lapse of time from starting at an extreme or other point to the arrival at the next intermediate or the other extreme point—the rate of travel

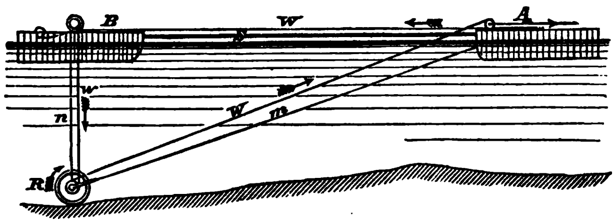
or speed being uniform throughout. The times, depths, and angles should be entered on the field-book with the greatest practicable despatch. The time for each 'sounding' is not required where the level of the water does not fluctuate. In reading the depths or soundings, if the surface of the water be agitated by waves, the range of the wave should be observed, and 'half-wave' noted for the sounding at the point.*

* It may be seen that the sounded point on the bottom is taken at random, and hence the doubts which attach to the representation of features obtained, without continuity, by these isolated points. To lessen the extent of doubtful representation, a great many points are sounded, which turn out to be afterwards useless for the purpose of the survey. In many instances, and from different causes, it will be impracticable to obtain the soundings and position of a sufficient number of isolated points to give a satisfactory knowledge of the features, even by considerable mental calculation. The same difficulty would be experienced by finding the level of dry land surface isolated points at random, which would give a very imperfect representation of the surface configuration. The superiority of contour maps for representing the features of the surface configuration is very decided, as compared with the representations of surface configuration by isolated soundings, or by a vertical section, along a particular line. The representations of the surface line on a vertical section is a more striking expression of features than figured dimensions to isolated points on the plan. For hydrographical purposes it would be difficult to make a contour survey of the bottom of a submerged district. To obtain a profile or vertical section is, however, a matter of little difficulty. In submerged districts with tolerably even bottoms, unencumbered with heavy marine vegetation, the section may be obtained with great expedition by a simple mechanical contrivance. The following description will make clear the features of the contrivance, and the mode of obtaining sections with it:—In the illustration (fig. 49), *B* is a small boat, and *A* a larger one. Into the bottom of the boat *B* is inserted a slot-box, coming above the water line, and open at both ends. *R* is a metal roller, with its axle projecting at both ends. Two forked nail-rod iron pulls fits on to this axle. The one is attached to a wire (*m*), a loop on which fits on to a hook in the stern of the boat *A*; the other forked wire-pull is

In districts where the level of the water fluctuates, two or more tidal gauges should be erected at suitable

attached to a fine brass wire (n), which passes over and round a sheave placed over the slot-box in B, and drawn by a weight over a pulley, so as to keep the wire straight between B and R. On the outer end of the axle of R is placed a small sheave, and a similar, but larger, one is placed on the end of a roller axle in the boat B. An endless wire (w) is made to pass round the large sheave in B, a block pulley, drawn by a weight over a fixed pulley in A, and round the sheave on the axle of R. A roll of paper is placed on a small roller, with a friction strap, in B, and drawn on to the axial roller of

Fig. 49.



the w wire large sheave by the motion communicated through the endless wire, or small steel band, from the boat A and the roller R. On the axle of the sheave, for the wire n, is made a hunting screw, and on this is placed a corresponding nut, which carries a pencil tracing-point so adjusted as to make a trace on the moving paper, which should be suitably supported between the rollers. The towing-line (S) is attached to A, and worked by a winch handle in B, to adjust the distance $A\ B$, so that the wire n ($B\ R$) may be vertical.

Now, in rowing forward the boat A, the roller R will revolve, and give motion to the endless wire, or steel band (w), which, in passing over the sheave in B, will draw off the paper, and thus communicate to this tracing surface a corresponding motion, independently of time. The tracing-point attached to the nut on the axle of the n wire sheave will have a motion proportional to the depth, or length of wire passed over. As the tracing-point and paper are in contact, and have their lines of motions at right angles to one another, the combined motions communicated, as described above, will give on

points, and their zeros set to the same level by levelling on land with a level, as already explained. If this be inconvenient, and the gauges free from river influences at low water, the zeros of the graduated surfaces should be set to a few feet below this level.

In situations where it may be inconvenient to erect a stake on which to fix the graduated tidal gauge, a gauging of the rise and fall of the water surface may be obtained by mooring a boat to three anchors, with the attachments so arranged as to enable the keeper of the register to keep the boat over a particular spot. The depths may be read on a steel tape attached to a heavy weight deposited on the bottom held vertically under the stern of the boat. In this case the zero of the scale should be the bottom end of the tape line.

During the time the survey operations are going on, the height of the rise and fall of the tide and the surface agitations should be observed on these gauges for short and equal successive intervals of time. The heights, fluctuations, and times should be entered on a register diary kept for the purpose. The entries on the register should be reduced to low water, by taking the reading for the latter from the mean reading of surface agitation, or crest and hollow of wave, for each registered time. The surveyor, furnished with the times on the register

the paper a profile or section of the bottom. The scales are easily found from the ratios of the gearing trains. The section may be taken in the night or day time.

[The apparatus described above may be obviously improved in some respects. With or without the apparatus, the matter is submitted to the consideration of others, as a suggestion well worthy of the attention of gentlemen interested in hydrographical surveys. The success which has attended the use of the apparatus described above—in a favourable district, however—points to its great practical utility in surveys for many purposes.]

and the corresponding heights of the tide, should find from these, by proportionality, the height corresponding to the time of each sounding. The timepieces of the surveyor and registrar should be frequently compared, and their goings entered, for the time, on the tidal gauge register.

The following form of field-book will be found convenient for most hydrographical surveys, in which the soundings shall be taken with a 'lead' and 'line':—

No. of Sounding.	Time of Sounding.	Gauge ht. for time.	Sounding.	Reduced Sounding.	Objects observed.	Index error + 0° 0′ 04″ + Angles measured with a sextant.						Objects observed.	Remarks.	
	h. m.	feet				°	′	″	°	′	″			
41	8.30	6¼	13½	7¼	B to C.	54	50	25	32	01	30	C to D.	Face up.	Land eddy to west 1 mile per hour, 8.30; increase steady to 10.30; then velocity of current was found to be 2 miles per hour.
42	8.36	6	11¼	5¼		51	0	20	28	57	10		Sand, &c.	
43	8.44	6	12¾	6¾		52	40	15	46	20	0		Mud.	
44	8.51	5¾	12	6¼		47	44	10	22	45	10		do.	
45	8.57	5¾	12	6¼		44	20	10	20	10	20		do.	
46	9.04	5½	11¼	5¾		41	10	15	18	04	30		do.	
47	9.10	5½	11¾	6¼		37	50	0	16	12	40		do.	
48	9.17	5¼	7	1¾		33	5	10	15	44	10		Sand.	
49	9.23	5¼	6¾	1½	Line in direction of Macedon House, Co. Antrim (see map of shore).								do.	
50	9.28	5¼	6¾	1½									do.	
51	9.33	5	7¼	2¼									Sand & mud.	
52	9.40	4¾	7¼	2½									do.	
53	9.45	4¾	7¼	2½									do.	
54	9.50	4½	7½	3									do.	
55	9.56	4½	7½	3									Sand.	
56	10.02	4½	6¾	2¼									do.	
57	10.09	4	5½	1½		25	30	0	22	28	30		do.	
58	10.14	4	6	2		31	30	0	15	30	05		do.	
59	10.20	3¾	5¼	1½									do.	
60	10.25	3¾	4¾	1									do.	
61	10.31	3½	5½	2	C to E.	57	45	0	36	30	10	D to E.	do.	
		*		*										

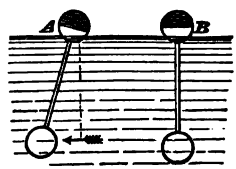
TIDAL CURRENTS.

In connection with the data obtained by soundings and the measurements described in the foregoing pages, it is sometimes desirable, especially for engineering purposes, to obtain a knowledge of the currents, their direction and velocity. For currents with a velocity of three feet per second or upwards, the current metre

* The entries, &c., in heavy type are obtained by reduction from the survey notes.

vane and compass card may be used. In currents with a lower velocity than about three feet per second, it will be found necessary to use a sensitive though less convenient indicator than that alluded to above. The substitute for the current metre, and which has found most favour, is shown in the illustration (fig. 50). *A* shows the axis of the metre bar inclined, under the action of a current indicated by the arrow. At *B* the metre is shown in still water, the axis being vertical. The float is hollow for the lower semisphere, the sunk globe or cylinder being connected with the float by a cylindrical rod. The upper part of the float globe should be made water-tight, partly filled with water, and covered with a scribed glass dome. When there is no current acting, as at *B*, the fine scale circles on the glass should be parallel to the enclosed water margin. When there is a current acting, as at *A*, these lines will be inclined to the enclosed water margin. The number of equidistant parallel circles intersected by the enclosed water margin will show the inclination of the connecting rod, and the corresponding velocity of the current. The circles may be made to express the velocity, which in this case may be read and noted directly, without interfering with the metre or taking it out of the water.

Fig. 50.



If floats be followed during a tide flow, and if their scribed domes and courses be observed at given times, the velocity and direction of the current throughout the tide flow may be found from the data obtained. And if metre floats be placed in the current at the same place and observed as above, the like velocity and direction at the place will be obtained.

CHAPTER V.

PARTICULAR SURVEYS—LEVELLING.

LEVELLING may be defined to be the art of measuring the difference of altitude between different points on the surface of the earth. When the points are on the same level there will be no difference of altitude. As allusions to, and the consideration of, level lines will appear frequent in what follows, it may be as well to define fully, at the outset, what a level line really is. *A level line is a perpendicular to the resultant of centrifugal force and the direction of gravity.* From this it may be seen that lines in a spherical surface, having its centre that of the earth, are, for all practical purposes, level lines. The surface of still water is a level surface, and points in it are on the same level; a line in this surface is, therefore, a level line.

That level line which shall be in the intersection of a particular spherical surface, and the vertical surfaces in which the surface trace of a line of section shall be, is called the *datum line*. The altitudes of points on the surface—the heights in the vertical surface of the line of section—have their zero in this line. The operations of levelling are conducted on the surface. The relative altitudes obtained should be combined with an initial datum distance (datum altitude), to obtain the datum distances for the several 'levelled' points on the surface. These reduced distances are usually entered on the 'level field-

book' in the column headed 'Reduced Levels.'* The datum line is usually selected so that the reduced altitudes shall be all positive, or, in other words, the altitude points all above it. This is convenient, but not absolutely necessary, as the datum line may be above all the altitude points, above some of those points only, or below all of them.

Levelling may be conducted in at least two ways: by the direct measurement of vertical heights—altitudes—and by the measurement of inclined lines and their angles of inclination. From this it may be seen that a levelling instrument, to aid in the measurement of altitudes by the latter mode, should have a graduated vertical limb, on which to measure the angles of inclination to the horizontal or level plane; and that in a levelling instrument for the finding of altitudes by the former mode, the requirements are reduced to the simple tracing of a level line. The best instruments available for those purposes are the *theodolite* and *spirit level*—'the level.'

The reader who is not acquainted with the construction and adjustment of these instruments should make himself fully understand what is given in Chap. VIII. before reading the following explanations of the art of levelling.

The practical object of levelling, in most cases, is to ascertain by measurements the features or surface configuration of a district. For this purpose measurements both in horizontal (level) and vertical lines are necessary. If the measurements be restricted to a single surface line,

* The expressions 'levelled' or 'level' point, and 'reduced levels,' are generally, but incorrectly, used instead of the *altitude point* and *reduced altitudes.*

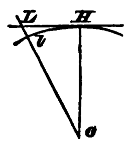
the features may be shown on a plane surface corresponding to the vertical surface in which the line is. This representation of features is called a 'section,' and sometimes a 'vertical section.' On this section, or representation, it is necessary to show the *datum line*, which line may be selected at any convenient distance from the centre of the earth, as already stated. If, however, the district, the features of which are required, be of sensible lateral width, of greater or lesser extent, the datum for altitude will be a corresponding spherical surface. In this case the features cannot be fully shown on a vertical section. The most expressive and convenient representation of the features of a surface will be hereafter described in 'contour surveys' (page 165).

The horizontal plane of a particular point touches the spherical (level) surface of the point at that point only. Hence, horizontal lines are not level lines except at the point of tangency with the spherical (level) lines. The line of collimation of a levelling instrument, when adjusted in the horizontal plane, is, therefore, not a level line. The trace of the line of collimation on remote objects is, however, by reason of atmospheric refraction, not strictly in the horizontal plane, but for distances in the horizontal of about 15 chains the density of the atmosphere will not sensibly vary, and hence the trace of the line of collimation to that distance may be, for all practical purposes, regarded, without appreciable error, as in the horizontal plane. And the line of collimation may be, therefore, regarded as a horizontal straight line touching the spherical or level line at the instrument. The deviation of the level from the horizontal line is, in levelling, the error due to the earth's sphericity. This should be deducted from the observed reading on a level staff to obtain the reading for the corresponding spherical

or level line. The errors due to the earth's sphericity at different distances may be found as follows:—

Let o (fig. 51) represent the centre of the earth, H the observer's station on the surface, HL the horizontal distance, Ll the *staff*. L is the point read from, H and l the true level point, Ll is the error due to curvature. Hl should be found by measurement. Then, from geometry $(2Ho + Ll)\,Ll = LH^2$. Taking the actual products, and rejecting Ll^2 as insignificant, we shall have, without appreciable error, $2Ho \times Ll = LH^2$, and $Ll = \dfrac{LH^2}{2Ho}$.

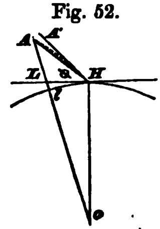
Fig. 51.



In this formula we have Ll proportional to LH^2. To find the relation between these lines in a particular case, the diameter of the earth $(2Ho)$ may be taken at 7,916 miles, and if HL be equal to one mile, we have by substitution $Ll = \frac{1}{7916}$. Expressing LH in feet, this becomes $Ll = \frac{5280}{7916}$ ($= \frac{2}{3}$ nearly). Then, to find the error due to the earth's sphericity for any other distance (d) expressed in miles, we have the corresponding error, $L'l' = d^2 \times \frac{2}{3}$ feet.

In levelling with a theodolite, the angle of elevation $A'HL$ (fig. 52) should be measured by observing the point A, and reading the vertical angle. HL should be found by measurement. This data contains two errors, which must be removed to obtain Al, the true difference of level. First, the ray coinciding with the line of collimation suffers appreciable atmospheric refraction, and lies along a curve, as shown in the figure. The tangent HA' to this curve at H is the true line of collimation. Hence the vertical angle is too great by

Fig. 52.



the angle AHA'.* This is the error due to refraction, which must be removed to find the true angle of elevation AHL ($= \theta$). The error AHA' will be $\pm$, according as the angle θ is below or above the horizon of H. Then, from trigonometry, $HL \tan \theta = AL$, and from above, $Ll = \frac{HL^2}{2Ho}$. Combining these we have $AL + Ll\ (= Al) = (\tan \theta + \frac{HL}{2Ho})\, HL$, the true altitude of A above H.

The angle H is only approximate, as the correction for refraction on the measured angle, for the fraction of the ray line near the earth's surface, which is the part used in these cases, cannot be accurately estimated. From this it may be seen that when a high degree of accuracy is required, the theodolite, as an angular instrument, is inferior to the level for levelling purposes.

If, however, the relative altitudes of two points equidistant from the station of the instrument be required, the difference in altitude will be truly found, independently of corrections for curvature or refraction, or like errors in the particular measurements.†

In levelling, the errors liable to be fallen into, independently of those already explained, and which may

* See Appendix, 'Table of Refraction.'

† If the distance HL be considerable—say 2,000 feet—and if l be a point on the still water surface of a lake, the error due to curvature may be computed, as already explained. If the instrument be in perfect adjustment, the difference between the true, or computed, and observed curvature will be the horizontal refraction between the points. The refraction varies at the same place, and with the angle of elevation being greatest at the horizon and vanishing at the zenith. It is so much less, for the short distances adopted in most levelling operations, than other unavoidable errors, that it may be altogether disregarded.

sometimes embarrass the engineer, may be classed as follows:—

I. Errors due to the level staff not being truly vertical.

II. Errors due to inexact adjustments in the field or imperfection in the construction of the levelling instrument, yielding ground, &c.

III. Errors due to a careless reading of the staff, or an inaccurate estimate for the subdivision of the graduations.

It may be observed in respect to these errors, that for equal distances the error due to imperfection of adjustment or construction of the level compensate, and do not appear in the results. For unequal distances the error will be approximately proportional to the difference of the distances. If equal distance points be made the carrying points for connecting the readings at different stations of the level, the resulting errors will only affect the intermediate independent and less important points. Except for great errors and instrumental imperfections, these may be disregarded in levelling for most practical purposes.

Errors due to the staff not being held vertical* are much more serious, as they may occur both at carrying and intermediate points. The error due to this cause is

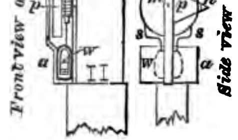
* To remove these errors, and expedite the levelling operations on a long line of levels, for water-works purposes, the author had a *tell-tale* constructed, to show when the staff was held inclined, and the error due to the inclination at the 11-feet joint, so that the error at the point of reading might be ascertained, and deducted from the reading before booking it. The illustration (fig. 53) shows a face and side-view of this *tell-tale*, and the point on the staff at which it was attached. The illustration is drawn to a scale. The *tell-tale* consists of (*a*) a japanned tin box, slotted on its upper side, and containing a weight (*W*), with disc rollers to keep it from bearing on the sides of the box *a*, when the staff shall be held up in use. A

uncertain and unmanageable, as the engineer can only judge of the vertical position of the staff by the common vertical plane in which the level and staff may be; but he has no certain knowledge of the inclination of the staff in that plane, and hence the errors so introduced are uncertain in amount.

The errors due to observation and yielding ground should be provided against by careful attention to reading with the side of the crosswire with which the

strong frame is soldered to the box, and carries on it a free disc or drum (m). On the arbour of this disc is fitted a small toothed pinion (p). The ends of a bent wire (s s) are inserted into the weight W, and a loop at its middle connected with a suspending link. A toothed sector is attached to the side wires s s, immediately below the pinion, when the weight roller is bearing on the bottom of the box a. To the frame is soldered an index wire (n), bent to appear horizontal on the face of m. The frame is secured by small screw nails to the side of the staff, and the weight is suspended by a fine wire from a pin at the 13-ft. division on the staff, so that the sector may gear into the pinion. The face of the disc m is painted red, white, and black from zero, to give the errors required. If the staff leans forward or backward, the weight W adjusts itself to the vertical under the suspending pin, and so gives motion to the disc m, the face of which is towards the engineer. This motion is proportional to the inclination, so that the distance from zero indicated by the index n enables the engineer to closely estimate the error, and, for all practical purposes, to absolutely remove it. The confidence thus given is very consoling, for the difficulty of finding a staff-holder to do his duty continuously and thoroughly is very great, whilst there is no confidence or assurance that the duty has been properly discharged.

Fig. 53.



TELL TALE

instrumental adjustment for level may have been made, and the movements of the body at the time of reading.

Instrumental errors, if any, should be removed by careful adjustment in the field each day before commencing the levelling. And the adjustments should be also examined in the field after concluding the day's operations, to ascertain if any disturbance occurred to vitiate the levels taken in the interval. [These observations do not apply to water and other self-adjusting levels.]

By comparing the errors alluded to above, it will be found that the error due to the earth's sphericity, without regard to refraction, will be $\frac{1}{8}$th of an inch for a distance or view of ten chains; and that the error at top of 'level staff,' due to holding it inclined 2° in the observer's vertical plane, will be about 0·01 feet irrespective of the distance. It may be observed that the difference between the distances of the carrying points seldom exceeds about six chains; and for an extreme distance to one of these points of ten chains, the error due to curvature would be about 0·01 feet. This would be equal to the error due to the inclination of the staff as stated above, but it would be less frequent. These errors are accumulative. That due to the curvature may be ascertained and removed, but the more serious errors, due to inclination of the staff, remain indeterminate. Hence the necessity for a 'tell-tale' contrivance attached to the staff to enable the engineer to observe and remove the errors due to inclination of the staff before making the entry on the field-book.

Levelling for a Vertical Section.—Levelling with the Level.

The problem to be solved in this service is to find the horizontal and vertical ordinates from rectangular

axes of co-ordinates, to intersect in corresponding points in the surface trace of the line of section. One of the axes of co-ordinates will be the datum line, and the other the vertical of the zero of the horizontal measurements. The altitudes, referred to the datum line, are obtained by observation on the level-staff held vertical. The horizontal distances are generally measured with the chain. If, however, the position of the surface points be ascertained on an inclined line, the measurements alluded to above as horizontal should in this case be made to the particular inclination.

The engineer should be provided with a good level, level-staff, chain, and arrows, and be attended by a party of four assistants trained to their respective duties. The division of the party should be, two chainmen, one staff-holder, and one level attendant.

Immediately before proceeding with the measurements for the section, the engineer should verify the adjustments of the level, as already stated.

The line of section being determined, and its trace shown to the staff-holder and chainmen, the former should hold up the staff on its end, or on a bench-mark,* in the vicinity of the end of the line of section, whilst the engineer should be engaged in setting up the level at a suitable point in or near the line of section, and making the field adjustments.

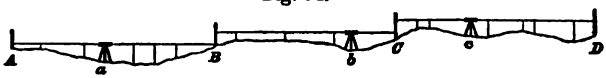
In the illustration (fig. 54) *A* represents the first point on which the staff is held in the line, and *a* the first station of the level. The telescope should be directed to the staff at *A*, the first backward point, the reading of which should be taken, and entered on the field-

* 'Bench marks' are well-defined points, which should be selected, by preference, on permanent objects, and fully described by sketches and measurements, so as to be easily re-determined, if required.

level book in the column of 'back sights.' After entering this reading, the verification of the field-adjustments and staff-readings should be observed and made correct before signalling to the staff-holder to move forward to the next level point in the line of section. This point being the intersection of the surface planes, is left to the immediate determination of the staff-holder, under the remote supervision of the engineer. The staff should be held on the points so selected, and the reading on it made, entered, and verified as directed above. If the point be an intermediate point between the first and last observed from a particular station of the level, the entry should be made in the column headed 'inter.' When the staff-reading for the last point read from a particular station of the level (such as B from a) shall be entered in the column of 'fore' sights and verified, the engineer should *not* signal to the staff-holder, as directed above, but carefully remove the level to a suitable forward station, such as b. After making the field adjustments for levelling at this station as at the former, the engineer should read the undisturbed staff on the last level point selected when the level was at the next former station, and enter this reading in the column 'back' sights, in which the staff-reading for the point A shall have been entered. In the meantime the chainmen surveying for the line of section and adjacent detail should keep pace with the staff-holder, and measure for the staff or level points, and report to the engineer the distances and measurements before he proceeds so far as to cause confusion in the sketching or booking of the detail or measurements. The engineer and his assistants should proceed with the measurements, &c., at the station b, for the view $B\,C$, and at all subsequent stations of the level, and their corresponding views in the manner described

for the views $A B$, $B C$ and stations a b. The points B, C, D, &c., for which there shall be a reading of the staff taken from the backward and forward stations of

Fig. 54.



the level, are called 'carrying points.' These are the points common to the readings taken at consecutive stations of the level.

Near the ends of the line of levels, as already stated, and at suitable intermediate points, 'bench marks' should be made on permanent objects, and their altitudes measured as points in connection with the line of section. These points need not be surveyed for the purposes of the section. Each day's levelling operations should commence on one and terminate on another 'bench mark.'

The engineer, before removing the level to a forward station, should reduce the 'fore' and 'back' readings, or measurements, to the *datum line* selected for the purpose. This reduction of the *levels*—altitudes—should be made in the field, at least for the carrying points. The operation will be readily understood from the following explanation, and an inspection of the illustration (fig. 54).

Let m and n be the readings of the staff at the carrying points A and B respectively, and d the altitude of A above the selected *datum line*; then will $d+m$ be the altitude of the line of collimation of the level at a above the datum line; and, therefore, $d+m-n$ $(=d_1)$ will be the altitude of the carrying point B above the same datum.

Putting m_1 n_1 for readings at B and C, observed from

b, we find the datum distance or altitude of line of collimation for this station of the level equal to d_1+m_1, and the like altitude of the carrying point C equal to $d_1+m_1-n_1$ $(=d_2)$. Stating the observed altitudes (levels), at length we find the reduced altitude for C thus: $d+m-n+m_1-n_1=d_2$, or $d+(m+m_1)-(n+n_1)=d_2$. This expressed in words is, *the difference between the sums of the 'back' and 'fore' readings added to the initial datum altitude* (datum altitude for first back reading point) *will be the datum altitude* (reduced level) *for the extreme forward point.* This, when reduced to a single station of the level, will be as above for d_1. By deducting the readings for intermediate points from the corresponding datum altitude of the line of collimation, the altitudes of those points above the datum line will be obtained. It may be seen that the difference between the 'back' and 'fore' readings is equal to the difference in the altitudes of the corresponding levelled points. This affords a convenient check on the arithmetical accuracy of the reductions for altitude.

In the operations just described there is not found a sufficient check on the results obtained, so that it becomes necessary to verify these by levelling between the 'bench marks.' The levels taken for this purpose are called 'check' levels, for which chain measurements and intermediate levels are not required. The levels for carrying points only may be taken, which points should be as near as may be at equal distances from the station of the level. As the check relates to the measurement of altitude, the absolute distance between the carrying points and the route selected will be, generally, immaterial. Should an error be detected between any two 'bench marks,' that part of the line of section should be relevelled in detail, and the point or points in error corrected.

The annexed 'Level Field-Book' will be found convenient and suitable for reducing in the field, without loss of time, the altitudes of the carrying points. The columns 1, 2, and 3 are placed on one page of a narrow

Belfast Water: L. Neagh scheme.

Levels between *B.M. at* 396·30

And *B.M. at* 429·90 *on line* (*Br.*).

Date *Dec.* 29, 1865.

	Inter.	Back.	Fore.
B.M. top stone in line, near fence		4·14	
In lea field	13·94		
In chain line	13·40		
	14·40		
	13·50		
Perp. 20	9·13		
Perp. 20			6·09
		13·76	
L.P. in chain line	6·76		
	2·85		
Middle co. road in line			5·02
		3·62	
In field	3·90		
Do. face of incline	5·25		
At tillage	8·75		
Cross fence	12·70		
Perp. 20			13·90
		8·03	
At fence in line	8·13		
	8·70		
Water	11·00		
x	10·10		
y	9·72		
Middle of bridge road	9·80		
Bed of stream (lower side)	16·30		
B.M.	7·37		
			10·64
		29·55	35·65
			29·55
		Fall..	6·10
	* 1	2	3

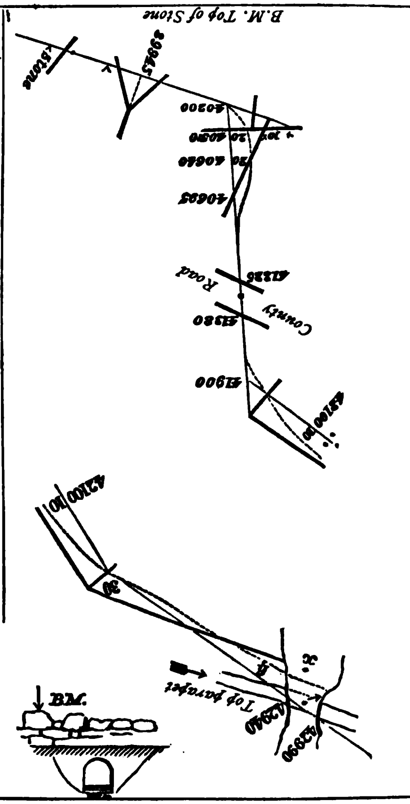
* The modern figures are those which the engineer should enter on the field-book, in the field, to record the measurements, and to obtain and verify the reduced altitudes (levels) of the carrying points. The antique figures are those which may be found subsequently in the office. Each levelled point, or reading, is on the same line as the corresponding reduced altitude.

blue-lined book, beside the columns 4 and 5, on the next, or face, page. The blue lines on the one page should correspond, in position, to those on the other, as the book, when open, makes only one page for the field work.

Engineer ______________

Assistants { ______________
____

Reduced Altitudes	Dist.
261·55	
265·69	396·30
251·75	396·45
252·29	398·00
251·29	399·80
252·19	402·00
256·56	405·10
259·60	406·40
273·36	
266·60	408·00
270·51	410·00
268·84	413·50
271·96	
268·06	415·00
262·81	417·04
254·06	418·12
231·36	419·30
258·06	422·00
266·09	
257·96	424·15
257·39	426·00
255·09	,, ,,
255·90	429·00
256·37	429·40
256·29	429·65
249·79	
258·72	
255·45	429·90
4	5



The following form of Level Field-Book has been adopted by many engineers :—

Belfast Water : L. Neagh scheme.

Levels between *B.M. at* 396·30
And *B.M. at* 229·90 *on Bridge.* Date *Dec.* 29, 1865.

Back.	Fore.	Reduced Levels.	Dist.	Observations.
		261·55 265·69	396·30	B.M. on stone near surface
4·14	13·94	251·75	396·45	In lea field
13·94	13·40	252·29	398·00	Chain 398·00
13·40	14·40	251·49	399·80	
14·40	13·50	252·49	402·00	
13·50	9·13	256·56	405·10	Perp. 20 left
9·13	6·09	259·60 273·36	406·40	Perp. 20 left
13·76	6·76	266·60	408·00	
6·76	2·85	270·51	410·00	
2·85	5·02	268·34 271·96	413·50	Middle co. road
3·62	3·90	268·06	415·00	In field
3·90	5·25	262·81	417·04	do. face
5·25	8·75	254·06	418·12	At tillage
8·75	12·70	231·36	419·30	Cross fence
12·70	13·90	258·06 266·09	422·00	Perp. 10 right
8·03	8·13	257·96	424·15	
8·13	8·70	257·39	426·00	
8·70	11·00	255·09	,, ,,	Water
11·00	10·10	255·99	429·00	Angle of field left
10·10	9·72	256·37	429·40	Middle co. road ; do. perp.
9·72	9·80	256·29	429·65	Middle of bridge
9·80	16·30	249·79	—	Bed of stream (lower side)
16·30	7·37	258·72	—	B.M. top corner stone of parapet, S.W. wall br.
7·37	10·64	255·45	429.90	
215·25	221·35 215·25	261·55		
Fall..	6·10	6·10		

It may be seen by an inspection of this form of Level Field-Book that to obtain the reduced altitudes (levels) of the carrying points, and their verification in the field, all the reduced altitudes must be found, and the sum of all the readings taken. Comparing the above form with the form of Level Field-Book already given, the advantages of the former are very manifest.

This form of Level Field-Book is well known, and was at one time very generally adopted by engineers :—

Belfast Water: L. Neagh scheme.

Levels between B.M. 396·30 and ______

Date ______

Back.	Fore.	Rise.	Fall.	Reduced Levels.	Dist.	Observations.
				261·55	396·30	B.M. on stone near fence
4·14	13·94		9·80	251·75	396·45	In lea field
13·94	13·40	0·54		252·29	398·00	Chain 398·00
13·40	14·40		1·00	251·29	399·80	
14·40	13·50	0·90		252·19	402·00	
13·50	9·13	4·37		256·56	405·10	Perp. 20 left
9·13	6·09	3·04		259·60	406·40	Perp. 20 left
13·76	6·76	7·00	0·67	266·60	408·00	
6·76	2·85	3·91		270·51	410·00	
2·85	5·02		2·17	268·34	413·50	Middle co. road
91·88	85·09	19·76	12·97	261·55		
85·09		12·97				
			Rise . .	6·79		
6·79	Rise . .	6·79				

All the figures for the reductions and verifications should be made, in the field, on this form of Level Field-Book, which requires six columns for figures.

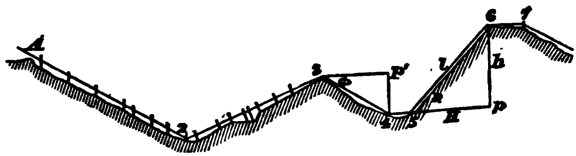
It may be observed that in this, as in the Level Field-Book immediately preceding, although there are several of the points intermediate between the carrying points, each intermediate point is entered as if it were a carrying point, which is not in fact the case.

Levelling with the Theodolite as an Angular Instrument.

The theodolite, as an angular instrument, may be used for the purpose of levelling, as already stated. The operation of levelling, in connection with the measurement of vertical angles, is as follows :—

The theodolite should be set up at the intersection of the selected inclined or other planes, such as at *A*, 2, 3, 4, &c. (fig. 56), and adjusted for observation. The index error of the vertical limb, if any, should be noted, and the limbs clamped at such angle as will make the line of collimation lie as near as practicable to the surface of the ground in the line of section. The vertical angle should be read, and the reading entered on the field-

Fig. 56.



book. Then a staff-holder and surveyor should proceed along the line of section, as from *A* to 2, or 2 to 3, and take up the feature points, or irregularities, of the surface. The engineer, at the theodolite, should signal the staff-holder, until the latter shall have adjusted the vane on his staff to the line of collimation, as shown by the cross wire of the diaphragm. The surveyor, making the measurement in the incline for the staff point, should at the same time enter the staff reading to the measurement for the point. In this operation the staff-holder may hold the staff vertical, as shown on line *A*—2; or perpendicular to the line of collimation, as shown on line 2—3. The manner of holding the staff should be stated on the field-book.

The angle ϕ and the line 3—4 are obtained by measurement. Then, to find the corresponding horizontal and vertical distances $3p'$, $p'4$, we have from trigonometry—

$$3p' = r \cos \phi \text{; and}$$
$$p'4 = r \sin \phi$$

A correction should be made to the latter for the earth's sphericity. As already explained, the actual path of the ray 3—4 cannot be rigidly ascertained, and hence the correction for curvature, or the earth's sphericity, will be only an approximation.

It may be seen that the operations with the graduated vane-staff is, in this case, for filling up minor details of surface irregularities in the operations for reducing surface to horizontal measure, already described in Chapter II. In this mode of levelling for a section, the trace of the surface will be obtained by plotting the staff readings on the line of collimation. This mode may be adopted, with some advantage, in rough country, when the altitudes are not required to be ascertained with more than approximate accuracy.

The theodolite may be used differently for the purpose of levelling, as a substitute for the level. When it shall be so used, it should be adjusted as for observation, and the vertical limb clamped when the bubble of the level tube shall be at the middle of its run. The horizontal limb should remain unclamped, to allow the instrument to turn freely on its vertical axis.

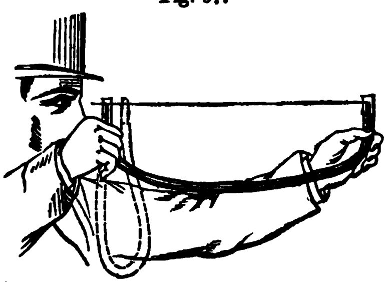
Other instruments may be used for levelling, but the range obtained with them is too limited to be of much practical service for general engineering purposes. The following are of this class:—

The Water Level—(Levelling).

This level is a self-adjusting instrument. It consists of a bent tube, filled with water. The surface of the water will be always in the same level. To facilitate tracing with the eye, the line in this surface at the ends of the tube, the ends are made of transparent glass. The prolongation of the line of sight touching the water

surface in the glass tubes determines this level on a remote object. The range at which levels may be accurately determined is obviously very limited, as compared with the range of the level or theodolite. Notwithstanding this limit of range for accurate levelling, the simplicity of the instrument and its self-adjusting properties makes it valuable for reconnoitring purposes, in which approximate levels may suffice. For this purpose the level may be made of a vulcanised indiarubber tube, with glass tubes inserted, water-tight at the ends. The glass tubes should be furnished with water-tight stoppers. In filling the tube, one end should be placed under water, whilst the tube should be lowered gradually, to allow the air to be fully expelled, and the tube filled with water. The ends should be placed together, held in the one hand, and in this position taken out of the water. If the ends be closed under water, with the stoppers, the tube may be lifted less carefully. When taken out of the water, the free ends of the level should be held vertical and close to one another, as shown in fig. 57. Then it will be seen, if all the air be expelled, that the water surfaces will stand at the same level. If there be air in one leg of the tube, the water surface will stand higher in that leg than in the other; when this is the case, the tube should be discharged and refilled.

Fig. 57.



By holding one end of the tube near the eye, and the

other remote, as shown in the illustration, the level of the observer's eye may be traced, approximately, along the hill sides by simply directing the line of the tube and view in this manner to the different points. The line so traced will be a contour. The eye may be readily adjusted to the level of any particular point by the observer moving up or down, until that point be observed in the level line of sight. The tube, when once properly filled with water, may be stopped at the ends, when not in use, rolled up, and carried in the pocket, ready for use when required.

The Reflecting Level.

This instrument is usually a plain glass mirror with parallel faces, supported on a stand and adjusted in the vertical by a suspended weight. It may be used for levelling at short range. The observer, in levelling with a reflector, should make the horizontal level scribe on the glass bisect the reflected image of the pupil of the eye, when the points of objects to be seen by reflection on the horizontal line bisecting the eye will be on its level. This instrument is better adapted for observing what points are on the same level than for following and determining the trace of a particular level line.

Levelling with the Barometer.

The principle for ascertaining the altitude with the barometer differs from that in the instruments alluded to in the foregoing. In those instruments the *altitude is not* represented in the level, which, by means of a liquid medium of uniform density, merely determines the horizontal plane of the point. In the barometer the *altitude is* represented, without regard to the horizontal

plane of the point. The barometer indicates the weight of the column of air above the point; and, as the height and weight of this column diminishes with the distance from the centre of the earth, the difference of the barometric columns for two given points or places correspond with the difference between the distances of these places from the centre of the earth. For levelling with the barometer, the column balanced at the level of the sea is generally taken as the zero for altitudes. If the barometric column for a given temperature at the level of the sea, and the barometric column for the same temperature at a known elevation above the level of the sea, be observed, the difference in the height of the columns will represent the altitude. And for any other difference in the barometric columns, at the level of the sea and a given place, the altitude will be proportional in the ratio determined as above. The indications of the barometer are more or less affected by the temperature; hence, for estimating altitudes, it is necessary to make a correction for temperature. The barometer should, therefore, have a thermometer attached to it for the latter purpose. The altitudes cannot be estimated with a high degree of accuracy, because of the difficulties to making accurate observation of the extremely minute subdivisions, as well as the liability to local thermometric or other disturbances, which interferes with the true indication for the altitude. For most levelling purposes it is not of much practical value; but for reconnoitring purposes, and more especially in mountainous country, it is a very valuable instrument, because of its portability and the independent manner in which the altitude of each point is obtainable.

TRACING CONTOUR LINES.—CONTOUR SURVEYS.

It may be observed that the height of the highest mountains are only a few miles above the level of the sea, whilst these mountains are several hundred miles from it at the nearest point; and that even the steepest mountains have their horizontal extensions many times greater than their altitudes above their bases. From this it may be seen that in an examination of the features, or surface configuration, of a district, a closer scrutiny is more important for the altitudes than for the horizontal extensions. This is recognised in the ordinary vertical sections for representing the features or trace of the surface line. In these sections, if the extent be considerable, and the features require to be closely examined, the vertical scale usually selected is much greater, and not unfrequently several times greater, than the horizontal scale. On contour maps the representation of height is made on the horizontal scale, so that the horizontal distance on the map, for a given difference of altitude, will be inversely proportional to the surface inclination. In this manner a very expressive comparative representation of surface inclination, or configuration, is obtained.

Contours are the trace on the surface of the ground of the level or *datum surface* already described. Contours are, therefore, level lines, which may be traced for all practical purposes as horizontal lines.

To simplify the consideration of the representation of surface configuration by contours, the reader should consider the vertical heights between the consecutive horizontal surfaces as constant. The contour surfaces should be selected at such distance apart as will make the

straight line, normal to their traces, be in the surface of the ground.

The water line of a lake is an illustration of a contour line. Now, if the level of the water be successively lowered by equal depths, the corresponding water lines will be contours at the same vertical distance apart. If the equal depths be so selected that the contour-normal line shall be in the surface, a map of the water lines will be a contour map, giving a very full representation of the surface configuration and inclination. If instead of a lake we consider the case of an island, on which the water surface would be raised by equal heights, the corresponding water lines would be the surface trace of horizontal sections of the island at the respective levels. This is another point of view from which contours may be considered. It may be seen that the greater the number of contours in a given altitude, the more minute will be the representation of surface configuration; and if the successive contours be the same vertical distance apart, the representation will be, graphically, the more complete. The greater the surface inclination, the fewer may the number of contours be for a given altitude.

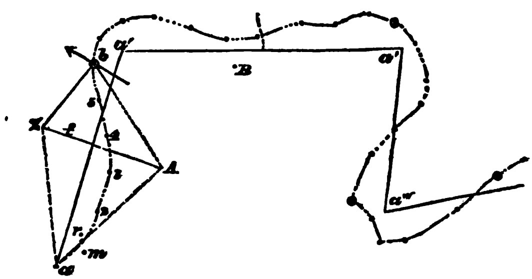
The instruments required for contour surveying are the level, chain, &c., box-sextant or prismatic compass, and the contour staff. The 'contour staff' is furnished with drum vanes and clamp screws, and in this respect it differs from the 'level staff.'

For the same station of the level the reading for the contour staff is the same for equal distances from the level. If the 'carrying point' distances from the level be equal, the reading for the contour will be the same for these and all intermediate points on stiff or steep grounds. On slightly inclined grounds the corrections for curvature should be made on the intermediate points.

From this it may be clearly seen that where curvature may be disregarded, the contour staff need not be read, if at each setting, or station, of the level a vane on the staff be adjusted to the plane of collimation of the level. By this means the range for tracing the contours is greatly extended. A brief description of the operations in the determination and surveying for contour points will suffice to make these simple measurements easily understood.

Let a (fig. 58) represent the ultimate known point in a contour trace, or a point from which it is required to trace a contour line, and let A be the first station of the level. After the level shall be adjusted as for levelling, the telescope should be directed to the level staff held

Fig. 58.



upright on a. The vane of the staff should be adjusted to the plane of collimation, on the level of the horizontal wire of the diaphragm, and securely clamped. Now, whenever the staff vane and the horizontal wire are on the same level, the staff will be on a contour point if the staff be held upright. From a the magnetic angle of $a\,A$ and $a\,Z$ should be measured, A being the station of the level, and Z a conspicuous object in the field. [If there be more than one conspicuous object in the field,

so situated that the lines from these to the contour points may contain an angle between 150° and 30°, some of these may be preferred to the station of the level for the purpose of laying down the position of the contour point.] From *a* the staff-holder should proceed approximately along the level line to the next succeeding place for the determination of a contour point. The selection of these points should be so made that the direct lines joining the successive points may be approximately in the surface. In field operations a skilful surveyor will select fewer points, so that a sketched line of feature through the selected points may be the true line. On the first setting-up of the staff, for the determination of a point in the contour, such as at *m*, the vane may be too high for the plane of collimation. This circumstance should be signalled to the staff-holder, who should accordingly take up a lower position, such as *n*, as near as may be in the contour. The relative levels of the staff vane and cross, or horizontal, wire of the diaphragm will show if the vane be still too high, too low, or on the level. In the case indicated in the sketch *n* is too low. This should be properly signalled or otherwise communicated to the staff-holder,* who will now perceive that the contour point is between *m* and *n*. The movements of the staff-holder should be determined and controlled in this manner until finally the vane be observed at the proper level, when the contour point 1 will be determined. From this point the magnetic angles should be observed for the lines from *A Z* intersecting in it. The contour points 2, 3, 4, 5, and *b* should be determined and surveyed in like manner. The last determined, or 'carrying,' point *b*, should be marked with a picket on

* A system of signals with a dog whistle will be preferred to flags, &c., &c.

the ground, to aid in a subsequent identification and verification of the trace, if desirable. After the determination, survey, and marking of the carrying point *b*, the level should be removed to a suitable (forward) place, such as *B*, and there set up and adjusted for levelling. The telescope should be then directed to the staff held upright on the carrying point *b*, and the vane on the staff at *b* adjusted to the plane of collimation of the level for the station *B* in the same manner as it was adjusted at *a* to the plane of collimation for the station *A*. The determination and survey of the intermediate, as well as the carrying contour points, should be proceeded with for this, and all subsequent ranges and their corresponding sections, as for the *A* range and section *a b*.

In the description given above, the determination and survey of the contour points are made simultaneously. This, although very desirable, is not absolutely necessary. If the contour points be marked on the ground, they may be subsequently surveyed by measurements with the chain or otherwise. The objections to the latter mode of obtaining data for the contour map are :—Great liability to omit, in the survey, some of the contour points, and the increased labour and delay occasioned by making the measurements. In some localities the trespass occasioned by increased number of hands has been found to be disagreeable, the more so as, from the curved (contour) outline, lines for the measurement of the determined contour points will be frequently selected, traced, and measured in places remote from the contour trace. The determination of the angular points of the reference lines so selected would occasion farther trespass if the lines be not selected to known points in detail, which, in many instances, would give an undesirable position for the contour detail measurements.

If the engineer have a good detail map of the district, to a suitable scale, the angles for the purposes of the survey may be measured to suitable, well defined reference points in the detail, which may be distinguished by figures, characters, or otherwise, as may be found convenient. The chain-measurements may be referred to the detail, where well defined, or to reference lines referred to well defined points in it. If there be not available a suitable detail reference map of the district, or if the detail be not sufficiently close, and the features be not so regular as to allow the measurements for laying down accurately the contour points, to be conveniently made, a traverse survey becomes necessary. The traverse may be run for the survey of the contour points with the chain, or for the survey of the conspicuous field objects, which latter may be temporary or permanent.

For the purpose of conveniently determining the points $A\ z$, the magnetic angle of the line $z\ A$ should be measured, and the line traced and measured to a convenient point, f. The line $a\,f$ and its magnetic angle should be measured. The line may be traced farther and measured to any convenient point a', from which (to verify the position of the point) the field objects of the survey, in view, should be observed for magnetic angles. In like manner the magnetic angles and lengths of the traverse lines $a'\,a''$, $a''\,a'''$, &c., and the position of field objects surveyed on them, should be determined. Instead of the magnetic angles for the traverse survey, corresponding angles may be measured with the theodolite, which will be preferable in most cases, particularly for long contour line traces.

If the vertical distance between the contours be less than the height of the staff, two vanes may be attached to it at that distance apart. One of these should be adjusted to

the plane of collimation for the particular carrying point before clamping the other at the contour distance. By this arrangement the feature points on the upper and lower contour may be very conveniently determined and surveyed as the traces proceed. The contour points in the level station range should be shown by sketches and measurements on a separate page of the field-book.

In proceeding to the contour survey of a district of considerable extent, the engineer should select certain easily accessible lines for preliminary reference or initial levels. These lines should be, if convenient, on the ridge dividing lines and valley lines of hills, which will be normals to the contours for feature representation on these prominent parts. Other 'initial' level reference lines should be selected to form convenient quadrilateral districts of greater or lesser extent, depending on the character of the surface configuration, and the degree of accuracy required in the survey. In close detail and cultivated country, the 'initial' levels may be taken on public roads, &c. Well defined 'bench marks' should be made for certain equal increments of altitude, and for certain equal horizontal distances in the line of *initial* levels, and at other suitable places, such as bridges, cut stone permanent erections, &c. These levels should be taken as for a vertical section, already described, and be reduced to the same datum. The position of the bench marks should be shown on a good detail map of the district, if such be available. If a detail map be not available, the lines of initial level should be 'traversed,' and the 'bench marks' indicated on a map laid down from the traverse survey. The level of each bench mark should be known, so that the trace of a contour may be taken up by levelling from one of them. The accuracy of the

distant trace of a contour may be verified by levelling from it to the most convenient initial level *B M*. From this it will be seen that the initial levels, by dividing the trace of a contour into sections, and by affording an easy means for taking up the trace and verification of the several sections, serve very important purposes.

For many engineering purposes contour maps give, perhaps, the most complete and convenient representation of surface configuration that can be well obtained. The superiority of this system of levelling, and the advantages afforded by the representations on a contour map, are fully recognised in towns, and on other valuable property. Its more general adoption is only a question of time.

The annexed contour map of a small district shows close contours and close detail. The blue lines represent the contours, which are figured to express the height above a particular level of the sea. This figuring of the contours affords assistance in the consideration of the altitudes.

Mining Surveys.

In mining, or underground surveys, the object will be to obtain complete knowledge of the form, position, and extent or capacity of the passages, galleries, headings, shafts, ways, &c. The position and general extent may be represented on plans and vertical surfaces or sections. In this service more measurements are necessary than in any of the surveys already described and explained, which were confined to measurements for the representation of one surface only.

From the essential peculiarities of an underground district, the survey must be made on 'traverse' reference lines. These lines will be, for the purposes of this survey,

not only the reference lines for horizontal measurements, but also for vertical measurements, and measurements in planes perpendicular to the reference line. The theodolite is the angular instrument best adapted for making the necessary measurements for this purpose. Instead of the ordinary tripod stand for the support of the theodolite, three low stools, or stands, should be provided. The stand or stool should be made with centre wooden screw pin, so as to serve the purpose of a suitable stand for the theodolite to screw on to. Two object lamps, with illuminated orbs, or bull's eyes, should be also provided. These lamps should be also made to screw on to the stand screw-pin, and to have in that position the centre of their illuminated orbs at the level of the axis line of the telescope attached to the theodolite. In addition to the lamps for this purpose, a small lamp with an illuminated orb should be provided and fitted to a staff, to slide on and be readily clamped to it. This staff or wand, and its attached adjustable illuminated orb, are for use in the trace of the traverse lines on the bottom, and vertical planes at intermediate points.

The survey operations may be commenced at any extreme point, and conducted along the several ways and passages to the other extreme point or points. These operations should be commenced by placing a stool or stand at the extreme point, for the beginning of the survey, and a stand at each of the two next forward angular points, for the traverse. On the first and third stands the object lamps should be placed, with their illuminated orbs towards the second stand, or first angular point. On this stand the theodolite should be placed and adjusted for observation. The surveyor should observe the bull's eye of the first lamp with the horizontal and vertical limbs clamped, and enter the readings of the vernier

stand, on which the forward lamp should be placed. The horizontal and vertical angles should be measured at the second, and all succeeding angular points as at the first. It may be observed that the stands only successively change the order of their position, the lamps retaining their relative positions throughout the traverse. The horizontal angles may be measured as for a traverse (overground) to a fixed line of direction, or independently.

When, at any angular point, the vertical angle for the forward line shall be 90°, the horizontal angle vanishes, so that the relative direction of the next succeeding reference line, making a vertical angle less than 90°, cannot be directly determined with the theodolite. The relative direction of such reference lines may be determined in two ways—namely, with the theodolite and compass by magnetic angles, and with the theodolite and a plane reflector. First, with the theodolite, &c., by magnetic angles:—

The compass card, or dial, should be accurately placed in the reference line near the theodolite, so that its centre or point of support shall be observed at the intersection of the cross wires of the telescope, when in the vertical plane of the reference line. After the card has ceased to oscillate, the magnetic angle of the reference line may be read with the telescope. The compass card and theodolite should be removed to the shaft extremity of the succeeding reference line with a vertical angle less than 90°. The compass card should be adjusted in the vertical of the shaft extremity of the last measured reference line by a plumb line, or otherwise, and its index made to read the magnetic angle for that line. The theodolite should be set up in the sight-line, of the compass card, or dial, at a distance from the latter,

and adjusted for observation. The telescope should be directed to the card, without unclamping the upper plate of the horizontal limb, and the bisection of its centre made by working the clamp screw to the lower plate. Now if the card reading be exactly the same as made at the former station, the vertical limb of the theodolite will be in the vertical plane of the measured reference line terminating in the shaft. The theodolite so adjusted will give the angles for connecting the lines. In this manner the reference lines of the survey for passages abutting on the shaft at one level may be connected with those of a like survey at another level, or with the reference lines of an overground survey. This assumes, however, that the magnetic variation does not vary. In many cases the assumption is not well founded, and in all it is open to question.

The second mode for connecting the reference lines of surveys for passages abutting on a shaft at different levels is as follows:—

Two points, remote from one another, should be accurately determined in the last measured reference line having a vertical angle less than 90°, which points should be distinctly and finely marked. Adjustable frames, stands, or stools, with lamps and plane muffled glass illuminated orbs, will be suitable for the purpose. On the illuminated orbs fine black rectangular straight lines should be drawn, and the adjustment made by placing their intersection in the vertical plane of the reference line. This should be done with the aid of the theodolite. A plane mirror, on an axis (in, or parallel to, its reflecting surface), should be placed and adjusted above the theodolite, and under the vertical shaft, so as to make the observed, direct and reflected bisections of the marked points identical. The axis of the mirror

will then be horizontal, and perpendicular to the vertical plane of the reference line, which plane intersects the mirror surface. The theodolite should be removed to the different level on the vertical shaft, and adjusted for observation. The telescope should be directed to the reflector, and the illuminated muffled glass orbs observed. If the theodolite be accurately in the trace of the vertical plane of the reference line, the reflected image of the black line intersection point will appear successively at the cross wires of the telescope, by making the mirror revolve slowly on its axis. If the reflected image of the marks do not appear at the intersection of the cross wires, the theodolite must be so placed, that when adjusted for observation the reflected images of these points shall appear accurately at the intersection of the cross wires. Then will the vertical limb of the theodolite be accurately in the vertical plane of the reference line traced at the different levels or altitudes. The trace of this vertical plane, in the districts at the different levels, or altitudes, affords the required connection between the reference lines for a continuation, or retrace, of the surveys at different levels.

Tunnelling.

In tunnelling it is required to determine underground passages which shall have known traces, and be at known depths below the corresponding overground surface. The survey operations are the reverse of those described in the foregoing for mining survey purposes.

At several points in the overground line vertical shafts should be sunk to the required depth, and the line of direction accurately traced on opposite sides of the shaft. This is usually done with the aid of a theodolite or transit

instrument. The practice of suspending weighted cords, or fine wire, from the points marked, to give the direction at the lower level, has obtained with many engineers, who adopt various means for prolonging the direction at this level. The practice, although convenient, is not reliable, particularly if the shafts be a considerable distance apart. The line of direction may be determined much more accurately, and with any desired degree of accuracy, by reflection, as already explained.

Identification and Determination of Detail from a Map of the District.

The identification of detail by map may be undertaken for either of two objects: the verification of the detail representation on the map; or the determination of the position of detail from the map-representation of detail on paper. The former service is usually called 'the examination,' and the party engaged on it 'the examiner.' The duties of the 'examiner' embrace more matters than the verification and correction of the position of detail on map. Those other duties shall be referred to in another place; in this place we shall only deal with the correct representation of the detail.

For the purpose of the examination, the 'examiner' should be furnished with an elegant and accurate trace (ink copy) of the plotted detail of the district, and he should be provided with a suitable sketch case (lined with prepared ass skin), pencil, lineal scale, chain, &c., and labourers. The trace copy, in one sheet, should be in extent not more than can be conveniently secured in the sketch case. It is desirable that the marginal detail on the trace copy shall be common to the adjoining sheets for examination. If the district be extensive, and

if there be more than one examiner engaged on the examination, adjoining sheets should not be given to the same examiner, that the character of the examiner's work may be ascertained by independent examinations of the same marginal detail. In the examination of the detail representation on a map the 'examiner' should be mainly guided by a few leading considerations; these are—

1. The position of a straight line, or detail, on the map will be correct when its actual and plotted position on the ground and map makes equal angles with another known line and intersects it in a known point, the position of which line and point on the ground has been previously ascertained to be correctly represented on the map.

2. The line, or detail, will be correctly laid down—given in magnitude and position—when its position and length on the ground and map are ascertained to correspond accurately.

From 1 and 2 it will be seen—

a. That the point of intersection of two given straight lines on the ground, and the corresponding point on the map, will be a given point on the map, if the corresponding lines on the map be ascertained to be correctly laid down in position. And,

b. That any two points being given or correctly determined, the straight line terminating in them will be a given line. Farther,

c. That a straight line traced* or drawn through given points is given in position.

* It should be kept in view that lines may be more accurately traced, and to a greater distance, with the naked eye, when the party tracing is rather above than below the level of the field on which the trace shall be made.

It may be also seen that a point on the map which is the common point of intersection of three straight lines drawn through well defined points in the detail will be a given point, if lines traced through the corresponding detail points on the ground be found to have a common point of intersection. And, farther, that the correct determination of two such points on the map determines, as already stated, the position of a straight line through these points. The determination, in the above manner, of three such common points of intersection correctly determines the representation of a given triangle. In the examination, the sides of the triangle determined by intersections, as above, should be measured on the ground, to ascertain and verify the accuracy of the determinations of the angular points on the trace or map. The production of detail lines, and lines traced through plotted points, should be taken up in the chain measurements of the sides of this triangle. Through these verified points straight lines should be traced, and drawn in pencil, to well defined points in the detail, such as the buttals of fences, the corners of houses and walls, gate piers, &c. On these lines the intersected and neighbouring detail should be examined by chain and scale measurements. In the measurement of the lines the internal and adjacent external detail should be very carefully examined, and corrected on the map where found in error. The examination of the detail should be carried forward by the production and intersection of given lines, and also by chain measurements from given points, to verify the position of the detail or other points on the map. This examination should be continued to the limits of the trace sheet. In remote parts of the trace and district, lines of verification should be drawn, traced on the ground and measured with the chain to verify

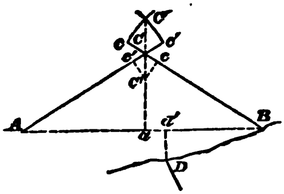
the scale measurements by the examination. These lines should be long, and in situations affording few facilities for the accurate determination on the map of the position of the plotted detail by other modes of examination.

The straight line passing through the extremities, or other well defined points in curved detail, should be regarded as a detail line, and the position of the intermediate curved detail verified by ordinates or tangents. Buildings and adjacent detail should be carefully examined by productions, &c., because of the greater difficulties these details usually present to the surveyor and plotter, and the consequent liability to small errors in the position of some of the plotted points, which affect the direction of lines determined on them.

In order to the correct determination on the ground of the limits of detail represented on a map of the district, two well defined points in the detail delineated on the map must be clearly ascertainable by inspection on the ground. And for a verifiable determination three such points should be so determinable. From a scale and measured distances, between two known points on the map and ground, the scale of the map (in the direction of the line) may be readily found. If a third well defined point, not in the direction of the measured line, be determinable by inspection, lines from it to the extremities of the measured line should be also measured on the map and ground. These measurements will give the data for finding the scale of the map in the direction of the measured lines. They also give data for finding the inequality of the contraction or expansion of the map surface or paper. If one of the scales of the map, ascertained as above, be selected as the scale for measurements, the measurements in the other directions may require correction. The correction may be made as follows:—

Let AB (fig. 60) be the selected scale direction without error, A and B being the identified well defined points. Let C be another well defined point. The paper lengths of AC, BC should be measured with the scale for the direction AB. These distances should be laid down, from their respective points A, B, in the lines from these points intersecting in C.

Fig. 60.



Let the chain measurements for the scale distances in these lines terminate at $c\,c'$. Then will $C\,c$, $C\,c'$ be the error due to unequal contraction or expansion. If the errors be inconsiderable, perpendiculars to the measured lines should be erected at $c\,c'$ and traced to intersect in C'. Then will C' be the position of the third point by scale measurements of the map, and C the true position; $C'\,C$ will be the resultant error. The line $C\,C'$ should be produced to d, which will lay down the resultant error line. $C\,d$ should be measured on the ground, and the scale of the map in that direction ascertained. The map will, therefore, have two scales if it may have expanded or contracted unequally; the one a primary scale for measurements in the direction AB, the other a resultant scale for measurements in $C\,d$ and lines parallel to it. The magnetic angles of these directions should be measured, and also the angle $C\,d\,B$.

A triangulation diagram should be laid down on the map, and also the survey of detail on its reference lines for transfer to the ground. Through the angular points of the diagram on the map, lines should be drawn parallel to AB and $C\,d$, to intersect these lines, or lines parallel

to them, as axes of co-ordinates, for the measurements in which the scales shall have been ascertained. The origin of the axes of co-ordinates should be selected and the axes drawn. On these the corresponding proper scale measurements should be made for the several angular points of the diagram. The angles of the several main triangles should be also measured with as much refinement and accuracy as practicable.

The origin of the axis of co-ordinates should be determined by measurements on the ground, and the axes traced for the whole or any part of the district. [It may be here assumed that all the angular points shall be determined from the same axes of co-ordinates.] The proper measurements should be made in these axes, to determine the corresponding reference points of the ordinates intersecting in the angular point. From these points the ordinates should be traced with the theodolite, and the point of intersection determined and marked with a suitable flag-pole, or other conspicuous object. When the angular points are all determined in this manner, the angles at each should be measured to verify the accuracy of the determinations, and afford data for computing the lengths of the sides. The scale and computed lengths of the sides being known, the error by scale should be proportioned over each line.

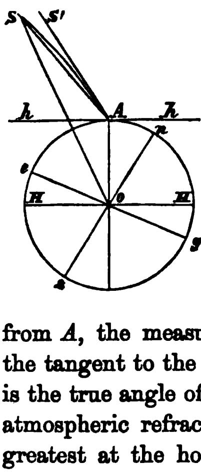
From the diagram and map a field-book by scale should be constructed for the measurement of the detail. The reference points, lines, and other measurements on the field-book so constructed, should be laid down on the ground, making proper allowance for the errors by scale, if any. In this manner the position of the detail represented on the map may be accurately determined. The measured points in the detail, by which the trace of the required detail will be laid down, should be marked on the ground.

LATITUDE.—MERIDIAN LINE.—LONGITUDE.

In many localities, particularly in out-lying districts and the colonies, it may be desirable to ascertain the latitude (lat.), longitude (long.), and meridian line for some point or points in the district. These may be ascertained and determined in various ways. It is believed the following modes will be generally found the most convenient and best suited for the place and circumstances.

Latitude.—In the annexed diagram (fig. 61) let $h\,h$ be the sensible horizon of the observer at the place A, $H\,H$ the true horizon, $n\,s$ the axis, and o the centre, of the earth, $e\,q$ the equator or zero of latitude, $e\,A$ the lat. of the place. Now the quadrants $e\,n$, $A\,H$ are equal; rejecting $A\,n$, we have $n\,H$ equal to $e\,A$ the latitude of the place. From this it may be seen that the measurement of the elevation of n will give the latitude of A.

Fig. 61.



If S be a heavenly body observed from A, the measured altitude will be $S'\,A\,h$, AS' being the tangent to the refracted ray $S\,A$. The angle $S\,A\,h$ is the true angle of altitude, and $S'\,A\,S$ the error due to atmospheric refraction. The error due to refraction is greatest at the horizon. It varies indirectly with the altitude, and vanishes when the latter is 90°, or at the zenith.* The angle $S\,A\,h$ is the true observed angle, and $S\,o\,H$ the true angle of altitude. The latter angle is greater than the former by the angle $o\,S\,A$,

* See Appendix:—'Table of Atmospheric Refraction.'

the angle of parallax, subtending *o A*, the radius of the earth. It may be seen that, like refraction, the parallax is greatest at the horizon and vanishes at the zenith, being inversely proportional to the angle of altitude.* It may be also seen that the errors due to refraction and parallax have opposite signs. The former should be subtracted, and the latter added to the observed altitude to find the true altitude. For a star the distance *o S* will be infinitely great, and therefore the parallax will be inappreciable.

The point in the heavens round which a fixed star apparently revolves is a point in the prolongation of the earth's axis. As the observer's place on the earth travels in the circumference of a circle whose radius of generation is a perpendicular to the earth's axis, the apparent path of the star, without taking into consideration the effects of atmospheric refraction, will be a circle similarly situated to that described by the station of the observer. Hence the centre of the apparent circle described by the star will be a point in the prolongation of the earth's axis. It follows from this that if the superior and inferiòr angles of altitude of the path of a circumpolar star be measured and corrected for refraction, one half the difference of the corrected angles added to the lesser, or taken from the greater corrected angle, will give the latitude of the place. The less the difference between the angles of altitude the less the error in estimating the refraction, and consequently the greater the accuracy with which the pole point will be determined.

In measuring the superior and inferior angles of altitude, the star should be observed, continuously, with a

* See table, 'Nautical Almanac.'

theodolite when approaching its superior and inferior altitudes, until the increase or decrease in the angle be found to cease; then the reading of the vertical limb will give the observed altitudes.

In southern latitudes this mode of ascertaining the lat. cannot be applied, the prolongation of the axis of the earth being depressed below the horizon. In those latitudes the meridian altitude of a known fixed star near the zenith should be measured and corrected for refraction; and the declination of the star should be taken from the Nautical Almanac and added to the corrected zenith distance to find the latitude of the place. As the lat. is equal to the zenith distance plus or minus the declination, the sign minus should be taken when the star is between the equator and the horizon. This may not unfrequently be the case in low latitudes.

If the meridian of the place be found as hereinafter described, the lat. may be found by observations of the sun, viz.: By measuring the meridian altitude of the upper or lower limb and correcting for refraction, parallax in altitude, and semi-diameter, to find the true meridian altitude of the sun's centre. The approximate longitude being known, the declination taken from the Nautical Almanac should be corrected for the place. Then lat. $=$ zenith distance $\pm$ declination. When the sun and place are on the same side of the equator, the sum of the zenith distance and dec. should be taken; but when on opposite sides of the equator, the difference of these arcs should be taken to find the latitude.

The Meridian Line.

The meridian line may be determined by observation on a circumpolar star as follows:—The theodolite, or small transit instrument, should be set up in a clear

open space, and adjusted for observation before the star reaches its greatest apparent eastern or western departure, or elongation. The star should be continuously observed with the clamps tightened, until the motion round the vertical axis be found to cease. Then the star will be at its greatest elongation on that side of the meridian. The verniers on the horizonal limb should then be carefully read, and the reading entered. Again, when the star is approaching its greatest elongation on the opposite part of its apparent circular path, the upper plate of the horizontal limb should be unclamped, and the telescope directed to the star. The upper plate should be again clamped, and the star continuously observed by operating with the slow-motion screws to the vertical limb and the upper plate of the horizontal limb, until motion round the vertical axis be again found to cease; then the star will be at its greatest elongation on this side of the meridian. The horizontal limb should be again read, and the reading entered. As the elongations are equal, the upper plate should be unclamped, and after being reclamped for half the recorded angle passed over, the vernier should be made to read this angle; then will the vertical limb be adjusted in the plane of the meridian, which plane may be now traced on the ground for the survey or other purposes.

The meridian line may be approximately determined without the aid of a theodolite, or other angular instrument, as follows:—

A flag-pole, or pole, should be erected on elevated ground, and a small lighted taper fitted on the top of it, when a circumpolar fixed star shall be approaching closely its greatest elongation from the meridian. The eye, at a low and remote station, should be brought into the line of the star and lighted taper produced by observing when

that $a\,b\,b'$ will be an isosceles triangle. The base $b\,b'$ should be measured and bisected in the point s, which will be a point in the meridian of a. The line $a\,s$ will therefore be the meridian line required.

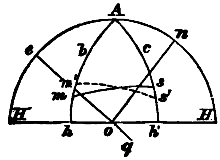
The trace of the meridian may be accurately determined in the day time as follows:—The sun's altitude should be taken with a theodolite about four hours before apparent noon, and the clamped horizontal limb read for the altitude. The sun's declination for the day should be taken from the Nautical Almanac, and the declination for the time found by proportionality. From this data the altitude for the afternoon observation may be also found without serious error. The vertical limb of the theodolite should be clamped at the computed altitude, and, in the afternoon, at the approximate time for the altitude the sun's course should be followed (without disturbing the lower plate of the horizontal limb, which should remain clamped) until the altitude be reached and the sun properly observed. Then the horizontal limb should be read, the upper plate unclamped, and the vernier adjusted to the bisection of the angle passed over. This will adjust the vertical limb in the plane of the meridian. From the station of the theodolite, the meridian may be traced on this angle, and other angles may be measured to connect the meridian line with the lines of a survey, or with lines for other purposes.

Longitude.—By Lunar Distances.

For finding the longitude by lunar distances, the angles of altitudes of the sun, or star, and moon, and their distance apart, should be measured with sextants, with the least appreciable lapse of time. These readings and the day and time of observation should be noted.

Let $m\,h$, $s\,h'$ (fig. 64) be the observed altitude of the moon and sun, or of the moon and a known star; also let A be the zenith, $H\,H$ the horizon, and $e\,q$ the equator; $o\,n$ the earth's axis, and $s\,m$ the measured lunar distance. The observed altitudes should be corrected for refraction, parallax, &c., to find the true altitudes $m'\,h$, $s'\,h'$, and their respective zenith distances $m'\,A$, $s'\,A$.

Fig. 64.



Now in the spherical triangle $A\,m\,s$ the three sides $a\,b\,c$ are given, and therefore half their sum s is given. From these data the angle A may be computed from the formula—

$$\sin \tfrac{1}{2} A = \sqrt{\frac{\sin\,(s-b)\,\sin\,(s-c)\;r^2}{\sin b \times \sin c}}$$

$$\text{or } \tan \tfrac{1}{2} A = \sqrt{\frac{\sin\,(s-b)\,\sin\,(s-c)\;r^2}{\sin s\,\sin\,(s-a)}}$$

In the spherical triangle $A\,m'\,s'$ the two sides $b'\,c'$, and the contained angle A are given to find $m'\,s'$, or a'.

By the aid of an auxiliary arc or angle, a' may be found as follows. Let θ be the auxiliary angle. Then

$$\tan \theta = \frac{\tan b' \cos A}{r}. \quad \text{And}$$

$$\cos a'\;(=m'\,s') = \frac{\cos b' \cos\,(c' - \;)}{\cos \theta}.^{*}$$

This gives the correct lunar distance by means of the cosines.

The lunar distances are given in the Nautical Almanac for intervals of three hours, mean Greenwich time. If

* Thomson's 'Trigonometry,' ed. 2, p. 32.

the computed lunar distance be found in the Almanac, the difference between the corresponding mean time and mean time of observation will be the longitude in time. If, however, the exact distance be not found, the difference between the next greater and lesser distances in the Almanac, corresponding to an interval of three hours, should be taken. The difference between one of these distances and the computed distance should be also taken. From these data the interval of time corresponding to tho latter difference may be computed. Calling the first difference d, the latter difference D, and the interval for this difference T, then

$$d : D :: 3 : T, \text{ and}$$
$$T = \frac{3D}{d}$$

If t be the Almanac or Greenwich time for the Almanac distance selected for finding the difference D, we have Greenwich mean time for computed lunar distance equal to $t \pm T$. Putting M for the mean time of observation, we shall have $(t \pm T) - M =$ the longitude expressed in time. If M be greater than $(t \pm T)$ the longitude will be east, and if less the longitude will be west.

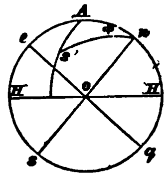
To Correct the Clock.

In the determination of longitude by lunar distances described above, it is necessary to ascertain astronomically the going of the clock. To the clock's corrected going the 'equation of time,' given in the Nautical Almanac, should be applied for the time of observation to find the mean time of observation. The latitude of the place should be known.

The altitude of the sun s (fig. 65) should be measured,

and the clock time of observation noted. The sun's declination for the day should be taken out of the Nautical Almanac, and the declination for the time of observation found by proportionality of the intervals of time and differences of declination, ascending or descending as the case may be. The observed altitude of the sun should be corrected for refraction, parallax in altitude, and semi-diameter, as already explained. From the corrected altitudes and declination the correct zenith distance $A\,s$ and complement of the declination $s\,n$ may be readily found. In the triangle $A\,s\,n$, θ represents the time of observation from noon, and $A\,n$ the complement of the latitude. The three sides of the spherical triangle are given. Adopting the usual notation, $s = \frac{a+b+c}{2}$, the angle θ may be computed from the formula for sine or tangent, page 191. By converting the angle θ into time, it will express the solar time of observation from noon. This should be corrected for the 'equation of time' for the day and time of observation, to obtain the *mean time* of observation, which is the clock's true going. By comparing the observed clock time of observation with the computed time, the error of the clock, 'too fast' or 'too slow,' will be ascertained.

Fig. 65.



PART II.

CHAPTER VI.

OFFICE WORK.

PLOTTING SURVEYS—MAP MAKING.

A MAP is a representation, on a plane surface, of the features of district to a diminished scale, as already stated.

The scale of the map, or plan, may be conveniently expressed as the numerator of a fraction, whose denominator shall represent nature. This expression of the scale is not true for the surface, but only for the lineal dimensions. Now if p represent the scale of a map, $n\ p$ will represent nature. Hence the ratio of the linear dimensions on the ground and on the map will be, in general terms, $\frac{1}{n}$. This is sometimes written 1 in n. If p be taken in feet, and n in chains, then, if $p = 1$, and $n = 30$, the expression for this scale is sometimes written '1 ft. to 30 chains,' or '2½ chains to an inch.' The latter and more inelegant mode of stating the linear scale is rapidly falling into disuse.

INSTRUMENTS USED FOR PLOTTING.

The instruments generally used in plotting are: Lines, of equal parts, for laying down linear and graduated

arcs, for laying down angular measurements. The former are called *plotting scales*, and the latter *protractors*. The primary and secondary divisions on the plotting scales are graduated to represent the corresponding primary and secondary divisions of the measuring instruments used in the survey. The angular instruments for plotting and measuring angles are graduated alike, as the angular, unlike the linear measurements, are not affected by the scale of the map.

The linear scales and protractors for sale are chiefly made of hard wood, bone, ivory, silver, brass, &c., and paper. The graduated sides of the scales and protractors are generally chamfered to a moderately thin edge, the graduations being cut on the chamfered surface. The best scales are those which expand least with heat, have perfectly straight edges, and are neatly and accurately graduated.

The following scales are required for plotting:—

No. 1. A 12-inch scale, graduated to scale of map, for plotting reference lines—3 (fig. 66).

2. A 2-inch scale, for plotting perpendiculars, and the pencil drawing of connections for detail—1 (fig. 66).

3. A diagonal vernier scale, for giving the measure of lines to be laid down by dividers, or compasses—2 (fig. 66).*

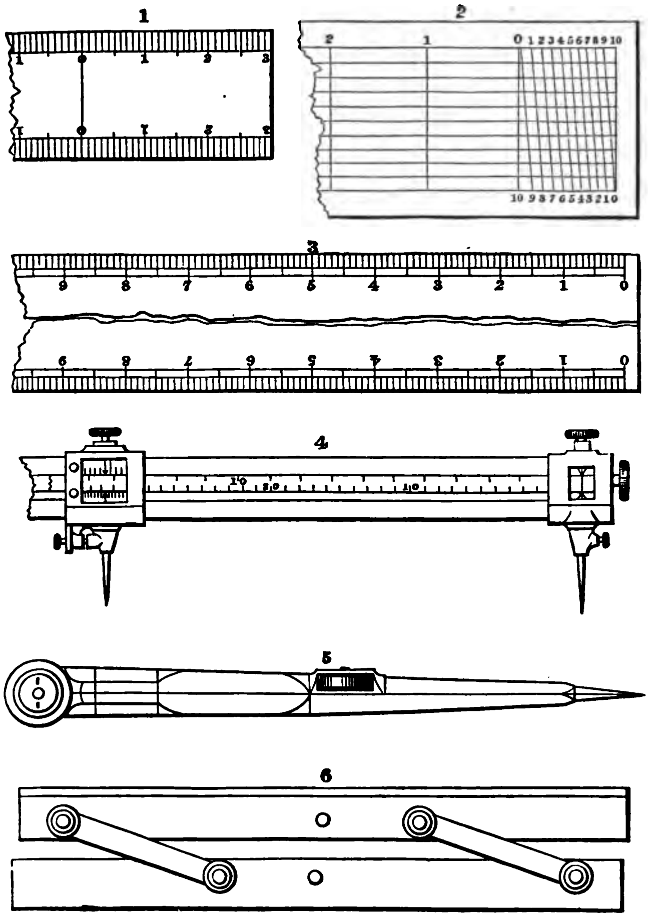
* The diagonal scale has the end primary division divided into ten equal parts, on two extreme parallel lines. Between those lines other parallel lines are drawn, so as to divide the distance between these lines into ten equal parts. A line is drawn from the first, second, third, &c., division on the one extreme line to the zero, first, second, &c., division on the other extreme line, respectively. The end subdivisions of the parallelogram space corresponding to the primary division are triangles, which are subdivided by lines parallel to the base. These parallel lines, in the small triangles, are pro-

4. A linear scale, with a slow-motion screw and vernier* arrangement for dividing the secondary divisions into minute subdivisions. The annexed illustration—4 (fig. 66) shows this scale and the minute subdividing arrangement. The following brief description will suffice in this place:—The zero end has a clamp screw and a slow-motion screw adjustment. The needle point is placed directly under the zero of the primary linear scale. The index, or zero, of the subdividing or vernier scale has its needle point adjusted directly under it, so that the needle points and the zeros of the scales shall be at equal distances apart. The vernier scale piece which slides on the beam is also furnished with clamp and slow-motion screw arrangement for making the required subdivision or reading on the primary linear scale. From its construction on a bar, or beam, this scale is usually called the 'beam compass.'
5. The (∧) compass, or divider—5 (fig. 66), is sometimes provided with a spring and screw arrangement, to facilitate the accurate adjustments of the needle points to the divisions on the scale.
6. A parallel ruler—6 (fig. 66), for drawing parallel lines.
7. An improved semicircular protractor (Hoskold's), with radial projecting arm and vernier, not illustrated.

portional to the corresponding divisions from the vertex, so that the difference between two consecutive parallel lines in the triangle will be $(=\frac{1}{10} \times \frac{1}{10}) = \frac{1}{100}$th part of the primary division.

* For a description of the vernier arrangement, see chap. viii.

Fig. 66.



8. A complete protracting circle, with verniers and extending arms.
9. A semicircular transparent protractor.
10. A paper, pasteboard, protracting circle.
11. A station pointer.

These instruments are made and sold by mathematical instrument makers. They do not require particular description, except perhaps No. 11. The station pointer consists of three radial bent arms, the outer two of which turn on a centre pin fixed in the third. The arms are furnished with platina wire, or hair, radial lines. Graduated arcs are attached to the side, or outer, arms; and verniers to the centre one. These are so adjusted that when the wires coincide, the zeros of the scales and indices of the corresponding verniers shall also coincide. The fixed centre is perforated to allow a pricker to pass through, and mark the centre point, or station.

The edges of all linear scales should be perfectly straight, and the graduations equally spaced and evenly cut. The equal spacing may be verified by taking in the dividers a given distance on the scale, and applying it successively to the succeeding divisions of the scale. The equal graduations on the protractor may be examined by measuring a given angle on different parts of the graduated arc.

In addition to the above-named instruments, good hard pencils, knife blades, indiarubber, set squares with straight edges, and a five-feet steel straight-edge, should be provided for office work. A straight-edge is known to be straight when a fine black trace drawn along it on a white even surface shall continuously coincide with the same edge directly applied to the other side of the traced line, without reversing the ends of the straight-

edge. In plotting, as in surveying, all reference and other lines for measurements should be perfectly straight throughout.

Matters requiring Attention.

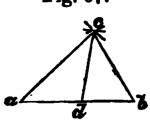
The office man should keep his hands and plotting scales clean, and avoid, as far as practicable, leaning on the fair sheet of paper. The fair sheet should be always kept covered with tissue paper, except when and where the measurements are being plotted. The pencil points should be always kept pared to a fine sharp conical point. The scales, when adjusted, should be held in their position by a light confident pressure of the fingers of the left hand. The eye should be placed in the line of the graduation on the scale, and trained to trace accurately its prolongation to the paper. The pencil point mark—plotted point—should be for reference points, as light and well defined as practicable. The plotted points for reference lines should be, surrounded by a small circle neatly drawn by the hand. The pencil drawing of the plotted detail should be fine, firm, and well defined. The pencil traces of reference lines should be directly between their reference points, well defined, and as light and fine as practicable. The degree of fineness and distinctness should be such that when the detail shall be finely, neatly, and firmly outlined, the reference lines and points may not be seen, except on a close inspection of the map, or paper, surface.

Plotting Triangles.

For plotting a triangle from the lengths of its sides only, a fine pencil trace should be drawn on the paper along the edge of a straight-edge. A reference point

'dot'—plotted point—should be made in this line, to which the zero of the plotting scale should be applied when its edge shall be parallel to and almost over the pencil trace, so that the zero line of the graduations produced shall intersect the reference point. The scale should be securely held in this position, and the scale distance for the length of a side plotted in 'the line' and in the production of the graduation; i. e. in a line perpendicular to the reference line at that extremity of the distance, which, if not a graduation, should be determined by estimation. The length for the side may be otherwise plotted by properly adjusting the needle points on a 'beam compass' to the reading for the measured distance, and laying the distance down directly by pricking at the same time with both points into the pencil trace. In the diagram (fig. 67) *a b* is the first plotted side of the triangle, *a*, *b*, being the plotted points. The split line reference point *d* should be in the first plotted side. This point should be plotted, as above, from the proper extremity of *a b*, and distinguished as a reference point. The lengths

Fig. 67.



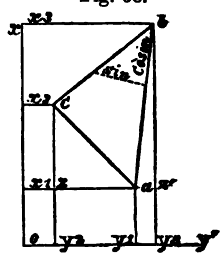
of the other sides, *a c*, *b c*, and the split line *c d* should be successively taken in the 'beam compass,' and scribed from their respective plotted points in *a b*, as centres, to intersect in *c*. The scribes so made will be found to have a common point of intersection if the several measurements be accurately made and correctly laid down. The lines for the plotted distances should be accurately drawn between their plotted extremities.

If the triangle be laid down from computed distances, in which case there may not be a split line, as the verification shall be otherwise obtained, the vertex point, or point of intersection of the side-scribes, may be otherwise plotted as follows:—[For this purpose the

measure of each angle should be known.] In the diagram (fig. 68) let $a\ b$ be the first plotted side, and c the scribe plotted vertex. The angles adjacent to $a\ b$ should be laid down with a protractor at their respective angular points, and their plotted lines produced to intersect. The point of intersection of the scribes should be found to be also the point of intersection of the lines by angles, if the field and office measurements be correctly made.

The angles may be more accurately laid down by means of a table of *natural sines* and *cosines*, and a linear scale, than with a protractor. The radius for the natural sines and cosines is taken as unity ($r = 1$). To find the sine and cosine for any other radius R, the sine and cosine in the tables should be multiplied by this radius; the product will be the sine or cosine required for the linear scale. Now, to lay down the angle b, the reduced cosine of that angle should be plotted from b in $a\ b$, with the scale of equal parts. From the extremity of the cosine, remote from b, and also from the point b, the scale length of the reduced sine, and also the scale length of R, should be scribed. A line drawn through b and the point of intersection of the scribes will lay down the angle corresponding to the sine and cosine in the table. The angle at a should be laid down in like manner, and the radius lines from a and b traced to meet in c, the vertex of the triangle. If the measured angle, or its supplement, be not found in the tables, the difference between it and the next greater or lesser angle in the tables, and also the difference between the next greater and lesser angles, should be taken, and also the table difference of the sines

Fig. 68.



and cosines. Calling the first-named difference d, the second D, and the last or tabular difference D', the corresponding tabular difference for d will be found from the proportion—

$$D : D' :: d : d', \text{ or } \frac{D' \times d}{D} = \pm d'.$$

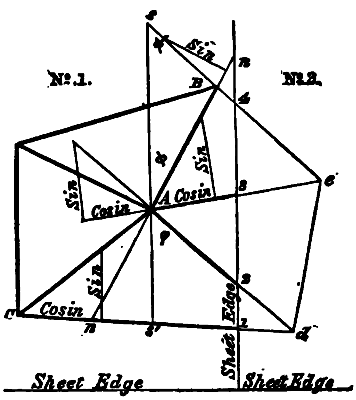
It may be observed that d' will be *minus* for the sines and *plus* for the cosines if, in finding d, the next greater angle in the tables be taken. If the next lesser angle be taken instead, d' will be plus for sines and minus for cosines.

If the latitude and longitude of the angular points be given for rectangular, or other, axes of co-ordinates, the triangle may be plotted as follows:—Let $o\,x$, $o\,y$ (fig. 68) be the axes of co-ordinates. These should be laid down from o, the origin, to contain the angle made by these axes. The distances in these axes, or lat. and long., for the points a, c, b, should be plotted from o. The plotted points will be $x\,1$, $x\,2$ $x\,3$, in $o\,x$, and $y\,1$, $y\,2$, $y\,3$ in $o\,y$. Through the points $x\,1$, $x\,2$, $x\,3$, lines should be drawn parallel to $o\,y$, to intersect in the points a, c, b, lines drawn parallel to $o\,x$, through the corresponding points $y\,1$, $y\,2$, $y\,3$. Lines joining the points a, b, c, will be the sides of the triangle, which should be found to measure on the scale the ascertained distance if the measurements and computations be correctly made and laid down. If the correct lengths of the sides be found by trigonometrical computation, the scale lengths should be found to correspond accurately with these distances, if the angular points be correctly plotted.

It will frequently occur, particularly in surveys of extensive districts, that all the angular points of some triangles cannot be plotted on the same sheet of paper. In this case the plot of the outlying points, and the sides of their triangles, may be laid down as follows:—

In the diagram (fig. 69) let n 1 be a common sheet-edge line and A, B, C, plotted angular points on sheet No. 1. It is required to plot the outlying angular points d, e, on sheet No. 2, and the sides of the triangles, intersected by the sheet edge, on both sheets. The straight sheet-edge line n 1, common to sheets Nos. 1 and 2, should be finely and elegantly drawn on both sheets. At A, B, and C,

Fig. 69.



the angles of the intersected triangles should be laid down with a protractor, or with a scale of equal parts, from a table of natural sines and cosines, as already explained. If convenient, the angles may be laid down on the opposite side of the angular points, as shown at A and B. The line $A B$ should be produced to intersect n 1 and $C d$ in n and n' respectively. The side $A n'$, in the triangle $A C n'$, should be found by computation, and $B n$ measured. Through A a line $s s'$ should be drawn parallel to the sheet edge, and the angles ϕ and θ measured with a protractor, and otherwise found by

sines and cosines. The angle θ is equal to the angle n. In the triangle $n\,n'\,1$ the side $n\,n'$ and the angles n, n', are known. The other sides $n'\,1$, $n\,1$, may be found by computation. The distance $n'\,1$ should be also found by scale measurement. In the triangle $s\,B\,A$ the angles θ and B and the side $A\,B$ are known, from which $A\,s$, $s\,B$, may be computed. The triangle $A\,B\,s$ should be plotted on $A\,B$, which will verify the laying down of the angle $A\,B\,4$. In like manner the triangle $A\,C\,s'$ should be plotted on $A\,C$, and $C\,s'$ laid down to verify the position of $C\,1$. In the triangle $n\,A\,2$ the side $n\,A$ and the adjacent angles are known; the remaining sides should be found by computation. In like manner the other distances from A to the points of intersection in $n\,1$, and the distances of these points from n and 1, should be computed to verify the plotted position of the intersected sides on sheet No. 1. On sheet No. 2, the points 1, 2, 3, 4, n, should be plotted from the adjacent sheet-edge point, and on 3 4 and 1 2 the triangles $3\,4\,e$ and $d\,2\,1$ should be laid down. The scale and computed lengths of $d\,e$ should be found to be in agreement. Then will the angular points d, e, and the lines intersecting in them, be correctly plotted on sheet No. 2.

If the lat. and long. of the angular or trig. points be known, and the axes of co-ordinates be parallel to the sheet edges, the lat. and long. of the angular points of the latter may be very easily found. The lat. and long. of the trig. points may be reduced to the angular point of any sheet edge by simply deducting the lat. and long. of the sheet angle point from the corresponding lat. and long. of the trig. points. The trig. points on each sheet may be plotted from the lat. and long. reduced to the sheet angle, selected as the origin of the axis of co-ordinates, which axes should coincide with the sheet edges intersecting in the selected angular point. If the same

angular point be selected as the origin for adjacent sheets, the sign of the lat. or long. will be positive on one sheet and negative on the other. By laying down the angles at the plotted trig. points, the position of the lines intersected by sheet edges will be correctly determined.

The plot on adjacent sheets may be differently made as follows :—The plot of the intersected triangles should be laid down, independently, and traced on tracing paper. The points on the trace corresponding to those already plotted on sheet No. 1 should be laid down on, and made to coincide with, those on the fair sheet. In this position the trace should be secured, and the sheet-edge lines traced on it. The intersected sides should be plotted on the fair sheet, with a pricker, at a point outside the sheet-edge line. The trace should be afterwards applied to sheet No. 2, and the traced sheet-edge lines made to coincide accurately with those on the fair sheet. The trig. points and intersected lines on sheet No. 2 should be pricked or plotted to the fair paper—the trace being securely held in this position.

Plotting Traverse Reference Lines.

The first reference line should be plotted as already described, and at the extreme point in which the second line intersects it the first measured angle should be laid down on the proper side of the plotted line. If the angles have been measured from a fixed line of direction, they may be all plotted at the first or any other angular point, care being taken to cipher, mark, or otherwise distinguish them. To each of these marks should be written the page of the field-book on which the survey of the line and measurement of the angle shall have been entered. In this case the plotting should commence at the angular point selected for the plotting of the measured angles.

The direction of each reference line so plotted may be transferred from the angular point selected for the plot of the angles to its position in the traverse by means of the parallel ruler. The directions of the lines should be consecutively transferred, and their lengths plotted from the angular points. The length of each reference line should be plotted in its proper place before transferring the direction of the next line to its position at the extremity of the adjacent plotted line. If the angles be measured independently, the plotting of the reference lines should follow the order and booking of the measurements, i.e. plotting the angles at their respective angular points. In the latter case, the angle or its supplement may be plotted as may be found convenient according to circumstances. The means for plotting angles has been already alluded to in the foregoing. If the angles be laid down with a protractor, its centre should be made to coincide with the angular point, when the 'back reading' on the graduations of the protractor shall coincide with a point in the plotted reference line or reference line produced, and a point plotted to the forward reading on the graduations to mark the direction of the reference line to be plotted from the angular point. In plotting the angles with protractors having rotatory arms and verniers, the latter should be set to the forward reading before laying down the protractor to the angular point.

The plotting of a traverse survey in which the angles have been measured from a fixed line of direction will be much facilitated by the use of a ⊥ square, straight-edge, and the semicircular protractor, with central perforation, radial arm, and vernier.* The straight-edge may be the

* It is believed Mr. Hoskold was the first to direct attention to this arrangement for laying down angles to fixed lines of direction.

side of a drawing or other board, on which the paper should be extended and fixed by pins or otherwise. By holding the head of the ⊥ square to the straight-edge of the board, at any required place, its blade will determine corresponding parallel lines; and by laying the straight side of the semicircular protractor to the side of the blade, so held, its zero will be adjusted to the fixed line of direction. The motion of the radial arm is not very free, so that it may be adjusted in the hand to the reading for the angle on the graduated arc, and the protractor, after being laid down on the paper, moved about and along the blade of the square to adjust its centre to the angular point, without disturbing the arm from its adjusted position. In this manner the angles may be plotted without producing the reference line.

If the lat. and long. of the angular points, with reference to axes of co-ordinates, be computed, the angular points may be plotted, as already explained, and the verification of the plotted points had by a scale measurement of the plotted lines. This is the only mode of plotting the reference lines of a traverse survey in which a check is obtained on the plotter's work.

Plotting the Secondary Reference Lines of a Detail Triangle.

The reference points of secondary reference lines should be plotted on the primary reference lines, or on one another, from the zero end of the line on which they may be surveyed, as already described for plotting the reference point of the split line. The scale distance between the plotted reference points of a secondary line should be found to agree with the measured distance, with an error uot exceeding 1 in 1000 in fair open

country, and $1\frac{1}{2}$ in 1000 in close country. Should these measurements differ by a greater amount than the above limit in the particular case, the straight line pencil trace should not be drawn between the reference points. The line should be entered on an 'office error sheet,' which should state the scale and measured lengths of the line, the number of the 'plot sheet,' the triangle, and the book, page, &c., in which the measurements are entered. Other lines referenced to the reference point or points of the line in error should be plotted, to ascertain if the apparent error be due to erroneous measurements for the reference points. The apparent cause of error should be entered on the error sheet; and, should the lines referred to the same reference point in error be found to plot to another point in the reference-point line, the scale measurement of this point should be also entered on the office error sheet. Further, if the reference point or points in error be not directly surveyed in the survey of their respective lines, the measurements for reference and the arithmetical reductions should be examined. If the error be due to an arithmetical blunder, the correction should be made without obliterating the original figures of the entry or the reduction. In many instances of apparent error, the measurements for the length of the reference line, and for the reference points, will have been accurately made though incorrectly entered. The points *f*20, *f*80, &c., are distinguished by the same marks on the chain, so that a careless reading may make the measurement for a point at the former read the latter. Errors of this class should not be corrected in the office, as the error may be due to some other unapparent cause.

In plotting reference lines for the verification of field

measurements, great care should be taken to have the divisions of the scale equally lighted, and to make the scale measurements for reference points from the proper end of the line, as already directed.

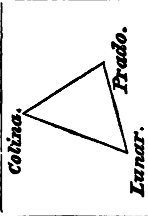
In addition to the 'office error sheet,' there should be an error sheet for each book and triangle, to be forwarded with the field-book to the surveyor for corrections. This sheet should give the book, page, and the line in error, together with some indication of the cause of error, without suggesting the probable amount.

[The corrections for errors should be entered on the field-book without erasures or obliterating the original notes, which should be cancelled with a stroke of the pen.] The field-book and error sheet when corrected should be returned to the office, and the points and lines in error replotted from the corrected measurements.

The subjoined are examples of office and field error sheets :—

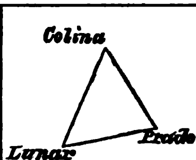
OFFICE ERROR SHEET.

Plotter's name ________________ Date ________

Book. / Page.	Lines.	Scale measurement.	Reference points of lines.		Apparent corrections.	Triangles.	Observations.
406 / 87	2775	2729	1940 / 2916	2373 / 4842	2473 / 4842		Examine reference point and line
406 / 42	3822	3878	3636 / 4842	2130 / 3904	3878		Examine line
406 / 57	2136	2145	2373 / 4842	907 / 3904	2473 / 4842		Examine reference point and line
406 / 24	3310	3330	2018 / 4842	2410 / 3904	reduc-tions		Additions in error—subtractions not made

FIELD ERROR SHEET.

To *A. B., Surveyor.* Date ________

Book. / Page.	Lines.	Triangles.	Observations.
406 / 37	2775		*Examine reference points and line.* $\frac{2473}{4842}$. Reference point (Jan. 4, 1864)
406 / 42	3822		*Examine line.* Line 3878 (Jan. 4, 1864)
406 / 57	2130		*Examine reference points and line.* $\frac{2473}{4842}$ (Jan. 5, 1864) { Corrected in the field, BALTA. HUERGA.

The plot for the verification of the secondary reference lines of each triangle should be made on a separate sheet of paper and preserved. The plot of the secondary reference lines on the fair paper for the map or plan should be made with equal care as that for the verification of the measurements of the reference lines. The plot on the separate sheet of paper should be made as soon as may be convenient after the survey of the triangle shall be completed. The plot on the map sheet should not be made until the detail of the triangle is about to be plotted.

PLOTTING DETAIL.

In plotting from the entries of measurements for reference lines or detail, the measurements should be laid down on paper in the order and manner indicated on the field-book. In plotting the reference and other points for perpendiculars and intersected detail, the long or line scale should be adjusted to the line and its extremity, as already described for plotting the line or

reference points on it. Several consecutive points, commencing at the zero, should be plotted in the line. The long scale should be then removed, and the small, or perpendicular scale, applied consecutively to the points, being reference points, in such manner as to make the zero of the scale coincide with the plotted reference point when its edge shall be perpendicular to the reference line. The detail point should be plotted on the perpendicular scale and on the proper side of the reference line. The proper connections for detail, as shown by the field-book, should be made by drawing a neat and firm pencil line along the edge of the short scale adjusted successively to the adjacent detail points. All check measurement should be applied with the scale before drawing the detail in pencil. In this manner the whole of the measured points in the reference line and the detail should be plotted, and the connections clearly and elegantly made. In drawing the detail, the sketching on the field-book should be looked upon simply as a guide for making connections, and not as a guide for features.

Before plotting the detail on a reference line, the plotter should carefully examine for the reference point, which shall be the zero of the measurements with the scale, and afterwards be careful to plot the detail on the proper side of the reference line. It is a cause of much annoyance to find, and of serious error if not found, detail plotted on the wrong side, or from the wrong extremity, of a reference line.

Plotting Vertical Sections.—Altitudes measured with the Level.

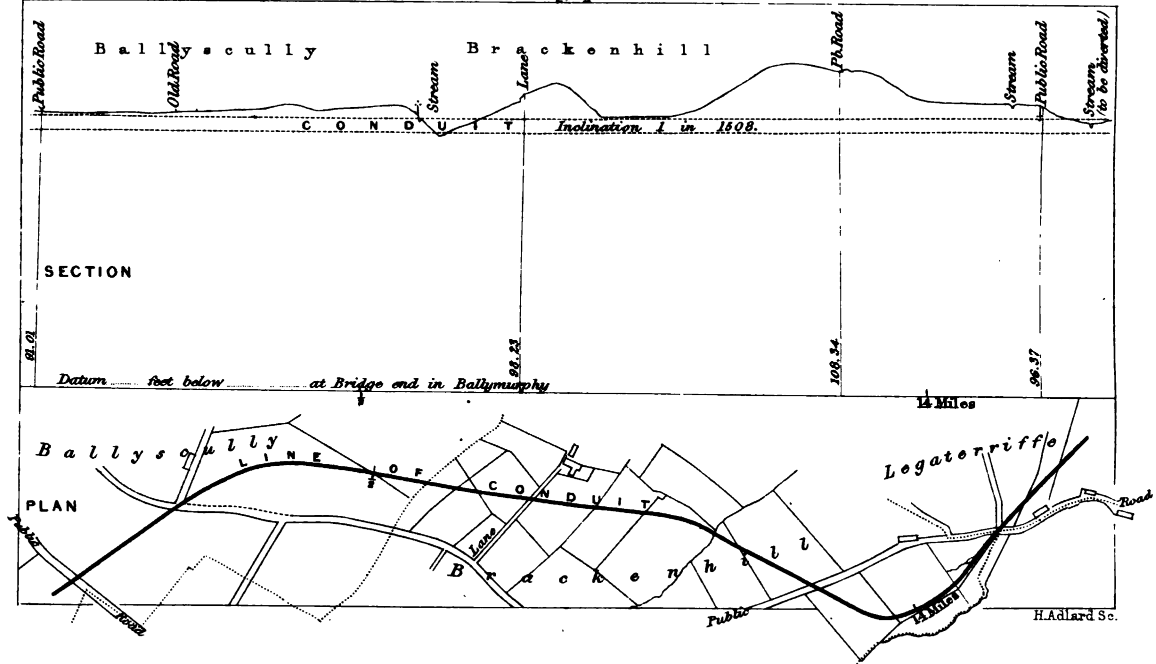
On the section paper a fine pencil line should be drawn along a 'straight-edge' to represent the *datum line*. On

this line the reference points for altitudes (or, in other words, the measured horizontal distances for level points) should be plotted with the horizontal scale from a zero, or other reference point, as already described for the plotting of reference points in reference lines. From these plotted points perpendiculars to the datum line should be drawn in pencil; and on the perpendicular lines the reduced altitudes of the corresponding surface points should be plotted on the vertical scale, with its zero adjusted to the reference points in the datum line. A line drawn through the consecutive surface, or level, points so plotted in the perpendiculars will be a correct representation of the surface line on a plane surface. The horizontal and vertical scales should be plotted and drawn in pencil.

For the purpose of erecting perpendiculars, rectangular set squares are made and sold: such squares are generally used in the plotting of sections by fixing a 'straight-edge' parallel to the datum line, and applying the one leg of the set square to it, whilst the other is made to be on the reference point, from which the perpendicular may be drawn along the edge of the set square so adjusted.

For most practical purposes a minute scale measurement of the horizontal distances is of less importance than may be desirable for like measurements of the vertical distances or altitudes. For this reason the vertical is, in some cases, taken greater than the horizontal scale. In those cases the surface line on the plotted section is a distorted representation of the true feature. (See illust. 71.)

Horizontal Scale 1 in 10560. Vertical Scale 1 in 720.



H. Adlard Sc.

London: Longmans & Co.

Plotting Vertical Sections.—Levelling with the Theodolite.

For levels taken with the theodolite as an angular instrument, the datum line should be drawn as already described, and the computed consecutive horizontal distances for the corresponding inclines plotted in order on it. From the plotted points perpendiculars should be drawn to the datum line. The several computed vertical heights, or altitudes, reduced to the datum line should be plotted on the perpendiculars from their respective reference points in this line. The consecutive altitude points so plotted should be joined by straight lines, which lines shall be the plotted reference inclined lines, or lines of collimation. The position and length of these lines may be verified by angular and scale measurements. The measurements with the chain for the several consecutive level points and their corresponding staff readings should be properly plotted on, and from, the reference line to obtain the feature points for the surface trace. A line drawn consecutively through these points will give the true representation of feature on a vertical section. An inspection of fig. 56 (p. 160) will make clear the manner of laying down the detail of surface feature.

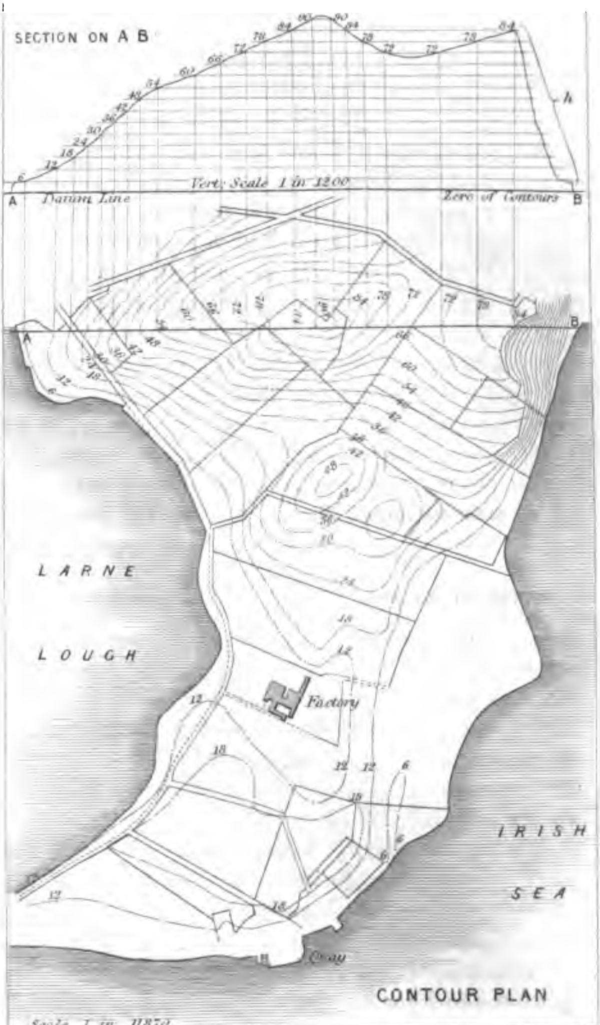
Plotting Sections from a Contour Map.

The contour map, for this purpose, should give the features of the surface configuration in sufficient detail without serious error. On this map the line of section, such as *A B* (fig. 72), should be drawn, and on the fair paper for vertical section should be drawn the datum line. The points in which the section line (*A B*) inter-

sects the contours should be measured on the scale of the map from a zero point in that line. The measurement, by scale, of the points in the contour should be plotted on the datum line on the fair paper for section. To the horizontal scale through these plotted points in the datum line, perpendiculars should be drawn, and on these the reduced altitudes of their respective contour points should be plotted. These latter points will be points in the surface trace. By drawing a line through the surface plotted points, the surface trace of the line or section will be correctly represented on the paper surface or section.

If the horizontal scale of the map and section be the same, the contour plane lines may be drawn on the section paper parallel to the section line on the map. If perpendiculars to the section line be drawn from the section-line contour points to intersect the corresponding contour or parallel lines on the section, the points of intersection will be surface contour points in a vertical section. Lines drawn through these points will give the surface trace of the line of section on a vertical surface. The section *A B* (fig. 72) is obtained in this manner. On the map the blue lines represent the traces of vertically equidistant contours. On the section the corresponding traces on the vertical surface, to the vertical scale, are also shown by blue lines. These are only for the purpose of showing fully the mode of obtaining the surface trace on the section. For practical purposes these parallel lines are only temporarily drawn in pencil until the surface trace shall be obtained, and drawn in ink together with a level or datum line. The perpendiculars from contour points on plan are also temporary pencil lines, drawn on the section for the purpose of illustration only.

The Horizontal lines h' on Section represent the equidistant contour surfaces (described as "drawn in blue,") and the Vertical lines from A.B. on Plan to intersect these (described as "drawn in black") should not be drawn on Section.



The Contour lines on Plan should be drawn in light blue fine broken lines.

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Plotting Contours.

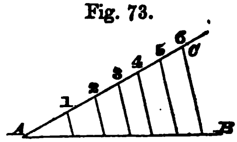
The reader who will have made himself fully acquainted with what is given in the preceding pages on the plotting of reference lines, reference and detail points, and the laying down of angles, will find little or no difficulty in laying down contour traces on maps, &c. If the contour points be surveyed with the chain, the contour will be obtained by drawing a free line of feature through the plotted points. If, however, the contour points be surveyed by the measurement of magnetic angles to known points, these angles should be laid down at the known points, and produced to meet in the contour point. In plotting contours, the square and semicircular protractor may be used with advantage. The carrying-points should be surrounded by a small hand-drawn circle, to distinguish it from the other contour points, which latter should be so plotted as to be distinguishable in the trace or contour line.

Plotting Sounded Points in Submerged Districts.

The plotting of the adjacent dry-land preliminary survey should be conducted as already explained: it only remains to explain the modes for plotting the sounded points. For angles measured with theodolites at dry-land points, the angles should be laid down at these points, and the lines produced to meet in the sounded point on the map. If the angles be measured on water, with a sextant, the arcs of the station pointer should be adjusted to read these measured angles. Then the pointer, without disturbing the angular adjustment, should be laid to the plotted points of the observed objects, so as to apply accurately the hair lines to their respective object points on the map. A prick into the map

surface, through the centre of the pointer, when it is so adjusted, will correctly plot the sounded point. If the measured angles be the angles made with the magnetic meridian (measured with the magnetic compass), the angles should be laid down at the plotted points of the observed land objects, and the lines, so plotted, produced to meet in the sounded point. Intermediate points in a straight line (the extremities of which shall be plotted as above) may be plotted when the observed data shall be the observed times of arriving at the sounded points, the sailing being uniform, by dividing the whole distance proportionally to the whole time and the observed intermediate intervals of time for the sounded points. This may be mechanically done as follows:—Let *A* and *B* (fig. 73) represent the extreme points determined by angles and plotted as above. From *A*, the first point in the order of time, a line *A C* should be drawn in any convenient direction, making an acute angle with *A B*. On *A C* the times of observation, 1, 2, 3, 4, 5, &c., *C*, for the soundings, should be plotted from *A*, with a scale of equal parts. Then *C* will represent the observed time for the determination of *B*. By adjusting the edge of a parallel ruler to *B C*, parallel lines should be drawn along its edge when adjusted to the plotted points in *A C*, to intersect and divide *A B* proportionally to *A C*. The points so determined in *A B* will be the required intermediate sounded points on the map.

Fig. 73.



Tracing paper, on which the measured angles shall be plotted to the same point, may be used as a substitute for the station pointer. The sounded point determined by angles measured with the sextant may be plotted by

describing circles on the land object lines as chords, to contain segmental angles equal to the measured angles. These circles will intersect in the common land object point and the sounded point. To plot the sounded point in this manner requires the problem, 'to describe on a given line a segment of a circle which shall contain a given angle,' to be solved by construction. This method is, therefore, less convenient than the other methods described.

Plotting Mining Surveys.

The plotting of underground district surveys does not require a detailed description in this place. The reader who will have made himself acquainted with what is given in the foregoing pages on plotting traverse surveys and theodolite surveys for vertical sections, will not find any difficulty in plotting on plan or section the survey of an underground district.

[In all cases and on all maps and sections, particularly on those for preservation, before proceeding with the plot of the lines or details the scale should be plotted near two adjacent edges of the fair paper.]

Copying Maps or Plans.

Any of the following methods for copying maps may be adopted according to circumstances :—

I. *By trace on transparent paper.*—The map or plan to be copied should be fully extended on a flat surface, face upwards. Over the map to be copied tracing paper should be kept fully extended by weights or otherwise. As the details on the map or plan will be visible through the copy paper, an accurate trace, in ink, of the details on the former may be directly drawn on the latter.

If it be desirable to obtain a copy on strong paper, the copy on tracing, or transparent, paper should be kept extended on the fair sheet by weights or otherwise. Between the trace copy and the fair sheet, before being so extended, there should be interposed light blackleaded tissue paper, with blacklead surface to fair sheet. By following accurately the details on the trace with a hard tracing point held nearly perpendicular, a correct copy, in blacklead, will be transferred to the fair sheet, which may be afterwards drawn in ink.

II. *By trace on the fair sheet.*—The map or plan to be copied should be extended on copying glass (glass table), over which the fair sheet should be kept extended by weights or otherwise. The copying glass should be so placed at a well lighted window, and the admission of light to the room should be so regulated, that the only light admitted to the apartment shall be through the copying glass, which glass should be nearly horizontal. To increase the illumination of the copying glass, a mirror may be placed under it to reflect oblique rays of light to it. The degree of illumination may, if required, be still farther increased by suspending in the rays of the sun a large glass globe, filled with clear water, to converge the sun's rays, which may be thrown by the reflector on the copying glass at the part requiring a powerful concentrated illumination. The light entering the darkened apartment through the map and fair sheet will make the detail on the former distinctly visible through the latter. By keeping the eye perpendicular to the

paper surface under trace, so as to avoid the introduction of errors due to parallax, the trace of the detail may be directly drawn in ink on the fair paper.

III. *By pricking through feature points.*—If the map be extended over the fair sheet, and secured in its place by weights or otherwise, the position of the detail on the former may be transferred to the latter by pricking through, with a fine needle point, the angular and other feature points in the detail to be copied. After the feature points are thus transferred, it will be necessary to draw the connections in pencil for comparison with the original map before drawing the copy in ink. The needle point, when used for this purpose, should be held perpendicular to the surface of the paper.

[The map copied will be injured to such an extent as to make this mode objectionable for the copying of highly finished or valuable maps.]

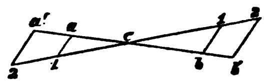
Reducing (Enlarging or Diminishing) Maps.

All maps of the same parcel or district, to whatever scale they may be laid down, are equiangular. It may be therefore seen that as the linear dimensions only vary with the scale of the map, mechanical contrivances may be constructed to aid in the rapid reduction of maps however complicated in detail. For reducing maps from one given scale to another given scale, the mechanical contrivance must have the tracing points for the maps in the same plane and straight line round an axis point on which they shall turn. They may be on the same or opposite sides of this axis. The following explanation

will enable the reader to understand more clearly the general principle of construction.

Let $a' b'$, 2 2, (fig. 74) be straight lines intersecting in c. If through 2 2 parallel lines be drawn to intersect $a' b'$ in a' and b', the proportion $2c : c2 :: a'c : cb'$ will obtain. And if $2c$ be to $c1$ in the ratio of the given scales, then will $a'c$ and cb be in the same ratio, the lines $a1$, $b1$ being parallel, to $2b'$, $2a'$. If $2c$, $c2$, be equal, then $a'c$, cb' will be also equal, and $b'c$, bc will be in the given ratio. It may be also seen that $b2$, $b1$ will also be in the given ratio.

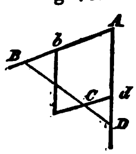
Fig. 74.



The mechanical arrangement must provide for keeping the parts corresponding to $a1$, $2b'$ always parallel, and the extremities of these lines in straight lines intersecting in the centre c. The following are of this class:—

The pantagraph (fig. 75) is a jointed rhombus, having two adjacent sides prolonged. The tracing points may be placed at B, D, on opposite sides of the centre C, or at B, C, if D be made the fixed centre. The opposite sides ($A b$, $C d$) of the parallelogram shall be always parallel. The fixed centre point C, in $C d$, and the tracing points B, D, in the prolonged sides are adjustable, so that they shall be in a straight line parallel to the diagonal $b d$, for all values of the angle A. In this instrument the similar triangles of the construction are $D C d$, $D B A$. If $A d$, $d D$, be made to have the given ratio, then $b B$, $C d$ will be each equal to $D d$, when $B C D$ shall be a straight line parallel to $b d$; and hence $B C$, $C D$, shall be in the given ratio. In this case, if one tracing point be placed at D, the other should be placed at B, and the fixed centre at C.

Fig. 75.

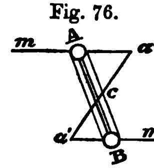


If however $A\,D$, $D\,d$ be made to have the given ratio, D should be made the fixed centre, and $C\,B$, the tracing points.

The points B, C, D, are known to be in the same straight line, and the tracing and centre points properly adjusted, when the steel tracing point, carried along a straight line of a given length, shall cause the pencil tracing point to draw a straight line, which should be found to be to the given line in the given ratio. The pencil point, when made to revolve in its socket, held fixed, will describe a circle trace on the paper if the tracing point be not in the axis of the socket. When the pencil point shall be adjusted truly in this axis, the trace on the paper for a revolution of the pencil in its socket will be a point. The pencil tracing point is usually provided with a pressure cup for small weights to keep the point on the paper. The number of joints and parts, together with the weight of this instrument, renders its use questionable, even in skilful hands, except for approximations.

The eidograph is a much less complicated mechanical contrivance than the pantagraph. It consists of a single bar $A\,B$ (fig. 76) fitted free on an adjustable centre c, which may be made to divide the given bar $A\,B$ in the given ratio. At the measured extremities of $A\,B$ are placed two equal drums, having their axes perpendicular to $A\,B$, and in the straight line $A\,c\,B$. Graduated arms $m\,a$, $m\,a'$ pass through slots, fitted with clamp screws, in the drums $A\,B$, and carrying sockets for tracing points at their extremities $a\,a'$. The drums carry a flexible steel band, with tightening screw adjustment for securing the points $a\,c\,a'$ in the same straight line, when $A\,a$ shall be to

Fig. 76.



$B a'$ in the given ratio--*i.e.*, $A a : A c :: B a' : B c$. When this is so, the bars $m a'$ $m a$ shall be parallel. The variable line $a' c a$ shall be also divided in the given ratio in the fixed centre c.

The verification of the parallelism of the bars $m a$ $m a'$, and the adjustment of the tracing and centre points to the same straight line, should be made as already described for the pantagraph. Like the latter instrument, the eidograph fails to give the reductions with sufficient accuracy for ascertaining distances from the reduced map for some engineering and other purposes.

If on opposite sides of a fixed centre, carrying a socket plate and clamp screws, two scales of equal parts, the graduations of which shall be to one another in the given ratio, be securely clamped on opposite sides of the fixed centre, with their graduated edges in a straight line passing through that centre, and also their zeros of graduations coinciding at the centre, the readings for detail on the one scale may be plotted on the other with as much accuracy as detail can be 'scaled' and measurements plotted. The detail plotted on the fair sheet should be drawn as on an original map. This arrangement, partly mechanical, is found in practice to be the most satisfactory and expeditious mode for the accurate reduction of maps.

Maps may be otherwise reduced by the construction of numerous similar figures, in the given ratio, on the map and fair sheet. The points of intersection of detail by the lines of the construction, and other points of the detail, may be determined by inspection on the corresponding lines and figures on the fair sheet. The detail drawn on the fair sheet to connect the points corresponding to the feature and other detail points of the map will be the required reduction. If the position and

measurement for detail be obtained, with a scale, on the construction lines of the map, and those measurements be laid down with a scale on corresponding lines on the fair sheet, which shall be to the other in the given ratio, the detail so laid down on the fair sheet will be more accurately reduced than when the reduction is made by inspection.

The similar figures of the construction may be squares or triangles. Laying down accurately the corresponding constructions is not altogether free from liability to error.

Verification of Plotted Detail.—Examinations.

The duties of the 'examiner' are to ascertain on the ground the correctness of the map representation made from the survey; to make the necessary corrections for omissions and detail in error; to give, in position and character, civil boundaries, &c., water, marsh, moor, vegetable and geological features, and permanent artificial erections; and to give also the descriptive names of places and things, or other desirable information.

That part of the duties of the examiner relating to the verification of the plotted detail has been already explained in connection with the identification of district by map, chapter V. The other duties are chiefly met by an inspection of the district and parcels, except in the determination of the position of isolated trees, which should be given in character. The topographical character of marshes, bogs, mountains, rough pasture, woods, water, &c., should be tinted and drawn in character on 'trace' (see illust. 77). If required, the hill sketching should be also supplied on the examiner's 'trace.'

Generally, the examiner should not make more than about three or four days' examination in the field before

'inking in' the corrections and omissions in red, and drawing the special features in character. The character shown on the illustration is that generally adopted for cadestral parcel and topographical maps.* These are:—Water, blue; Deep Bogs or Peat Beds, brown etching and blue lines; Marshes, reed and rush etching and blue lines; Rough Pasture, black etching; Woods, Underwoods, &c., as shown; erections in masonry, red; Boundaries as described on illustration. The descriptive names should be elegantly written, in character, on parallels of latitude.

On the examiner's trace being returned to the office, the plotter should replot from the field notes of the surveyor the detail corrected, and seek the cause of error. If the error be one in the plotting of the detail, the latter should be plotted from the surveyor's field-book. If the error be due to defective or erroneous measurements, the corrected detail should be plotted on the fair sheet from the examiner's 'trace.'

Map Drawing.

The detail sheet to be drawn should be placed under a full, but not diffused light, and kept as clean as may be practicable. The materials and drawing instruments should be suitable for the purpose. Besides these, the draughtsman should be provided with the graduated scale for the map, the surveyor's field-book and the examiner's corrected 'trace' copy of the plotted detail, pencils, &c.

Before proceeding to draw in ink, all corrections on the examiner's 'trace' should be examined by the draughts-

* The delineations of character which should be shown in the colours referred to are printed in black from engraving.

man, in connection with the surveyor's field-book, so that the final plot of the corrections shall be properly made or verified by him. All details on the examiner's 'trace' should be transferred to the map by plotting before drawing in ink the surrounding detail, in order that the detail be plotted before any of the reference points shall be obliterated.

The drawing-point selected by different draughtsmen, and classes of draughtsmen, requires some attention. Some prefer a sharpened steel pen point, others prefer a crowquill pen point, whilst many, and those the least skilful, prefer the steel bow pen for drawing the detail outline on a map or section. Of these, the steel pen point, properly ground and skilfully used along the edge of an ivory scale, will give a finer, deeper tinted, and more elegantly finished outline than any of the other line drawing points or instruments referred to. In drawing outline with a fine, or ground, steel pen point, skilful draughtsmen proceed continuously along the pencil-drawn detail by partly repeated touches with the pen point, so held to the scale edge, that the divided points of the pen may follow partly in the same track. Each touch is generally made about one-thirtieth of an inch in length, the touch beginning and ending fine, and advancing the line by rather less than half that length. Lines, whether straight or curved, when drawn in this manner, will be uniform in depth of tint and thickness of line. The junctions of detail are shown clearer, sharper, and better defined than these parts are drawn with the other line drawing points, particularly the bow pen. For many purposes, particularly for rough drawings, the bow pen is the better line drawing point. The dots for representing undefined detail should be longish and level throughout.

The representation of vegetation and other like super-surface objects shown on the examiner's trace, should be drawn in character, as represented on the annexed map (illust. 77). On plans, or maps to a small scale, it is generally preferred to give the bold representation of leading super-surface features in elevation, attending at the same time to the characteristic of individual classes of vegetation. On plans to a large scale, the super-surface features are generally drawn in plan. Boundary and other lines (see illust. 77) should be drawn as shown on the examiner's corrected 'trace.'

Letter-writing should be elegantly executed in lines parallel to the top or bottom of the map sheet edge. The sheet edge lines should be drawn straight, firm, light, and well-defined.

The meridian line should be drawn on the title page map, to show the angles it makes with the sheet edges. The scale should be drawn, on each sheet, where plotted.

HILL SHADING.

Shading for the representation of surface configuration is not advisable, except on geographical maps to a small scale, on which, with some important exceptions, artificial detail is generally omitted. Two principal devices for representing surface configuration by shading have been more or less adopted. The older, more easily drawn, less reliable, and less expressive, consists in drawing short lines, approximately normal to a horizontal in the surface, of depth and closeness proportional to the inclination of the surface. These lines represent the surface water lines, which accurately indicate the direction, but not the degree, of inclination. The light is supposed to be vertical; as, however, the observer never

Scale: 1 in 10560 ($\frac{1}{10560}$)

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sees a physical surface of considerable extent under a vertical light, the surface shading is generally drawn as for an oblique light. The accuracy of the representation of surface configuration is thus farther vitiated, as for the same inclination, there is a difference in the depth of shade depending on the position in relation to the light. The more recently adopted and expressive device for the representation of feature may be called contour shading, which consists in drawing continuous horizontal lines of equal depth, the number of such lines being made to represent the angle of surface inclination. The map would be a hand-sketched contour map, with a fulness of expression depending on the greater number of lines to express the inclination of a given surface. Surface shading by the latter device, although much more expressive than the former, requires greater skill in the execution, as a false feature will be given by a careless drawing of the horizontal lines; but no such false expression will be given by a like drawing of the short broken lines, and hence the insufficiency of the older device for the representation of surface configuration.

Drawing Sections.

The datum line, which may be that drawn in pencil, or a line parallel to it, should be drawn a fine, well-defined, deep carmine red line. The vertical lines should be drawn in brown, and the surface line in black. The (new) surface line for contemplated works should be shown, if in black, by an undefined line; if in red, by a defined line. These lines may be seen engraved by an inspection of illust. 71, which shows a plan and corresponding section with line of projected works.

It will depend on the use to be made of the section to

what extent the datum distance of level points should be expressed by figures written on their respective vertical lines. For working drawings, each reduced altitude, or datum distance, is generally figured on their respective vertical lines on the section. A short description of remarkable or important surface points should be written to them to facilitate identification and inspection.* The horizontal and vertical scales should be neatly drawn and fully described in writing.

Drawing Contours.

The drawing of contours differs somewhat from drawing ordinary detail, in so far as each contour point should be shown by a small black dot, and each carrying point by a like dot surrounded with a small circle in black. The line joining adjacent points should be true lines of feature. These lines should be drawn in light blue, and of a broken or slightly undefined character for representation. The contour lines should be readily traceable, but not conspicuous. (See contour map, illust. 72).

In the continental countries, which have adopted the French topographical system, contour lines are generally drawn continuous, a broad but faint line of colour.

* Sections of works for Parliamentary purposes require to comply with 'Standing Orders' of both the Lords and Commons, which are altered from time to time.

CHAPTER VII.

OFFICE WORK.—COMPUTATIONS.

[NOTE.—In writing the foregoing pages, it was judged more advisable not to break the continuity of the descriptions by the introduction of arithmetical computations, or the frequent discussion of mathematical formula. The discussion of general mathematical formula should be sought in mathematical books for the purpose, in which the details of the proofs may be found. The needless multiplication of arithmetical examples, to meet every possible case in the application of formula to practice, would not be an advantage. It is believed the reader who will have made himself fully understand what is given in this Chapter will find little or no difficulty in approaching and conducting computations for all other cases, or applications, of the formula. As many of the computations will be made with the aid of logarithms, there will appear, occasionally, notes and observations unnecessary for those fully acquainted with the construction and use of logarithmetic tables.]

INCLINED PLANES, &C.

1. To find the horizontal distance and altitude, or vertical height, corresponding to a given angle of inclination and a given distance or length on the incline—

Let l (figs. 1, 56, pp. 13, 160) represent the distance on the incline,
h the corresponding altitude or vertical height,
H the horizontal distance,
And θ the angle of inclination.

Now from trigonometry for the radius 1, $H = l \cos \theta$; and $h = l \sin \theta$. By supplying the radius r for logarithms these become—

$$H = \frac{l \cos \theta}{r}; \text{ and } h = \frac{l \sin \theta}{r} \text{ respectively.}$$

The logarithmetic expressions for these formula are—

$$\log H = \log l + \log \cos \theta - 10, \text{ and } \log h = \log l + \log \sin \theta - 10.$$

For a particular application of the formula let $l = 400$ and $\theta = 14° 28' 40''$. Then by the formula for the radius 1—taking natural sines and cosines—$H = 400 \times$ nat cos $14° 28' 40''$, and $h = 400 \times$ nat sin $14° 28' 40''$.

nat cos $14° 28' 40''$ (from the tables) $= 0{\cdot}96806$.
nat sin $14° 28' 40''$ (from the tables) $= 0{\cdot}25001$.
And $0{\cdot}96806 \times 400 = 387{\cdot}224$, the horizontal distance H.
Also $0{\cdot}25001 \times 400 = 100{\cdot}04$, the altitude h.

Taking logarithms.	To find H.	To find h.	
log 400 $(= l) =$	2·6020600	log 400 $=$	2·602,0600
log cos $14° 28' 40'' =$	9·9859851	log sin $14° 28' 40'' =$	9·397,9478
	12·5880451	$-\log r$ $=$	10·
$-\log r$	10·0000000	log 100·002 $(= h) =$	2·000,0078
log 387·3 $(= H)$	2·5880451		

NOTE.—The log. radius 10 may be subtracted by deducting 1 from the second place of the index of the sum of the logarithms without writing out the operation in full.

2. Tracing straight lines past obstructions. Finding inaccessible distances. (See illustration 3.)

To find the point a'—

$$\text{Formula.} \quad P a' = \frac{d' a \times P P}{d d'} \text{ (p. 24).}$$

Let $d' a = 60$, $PP' = 300$, and $d d' = 100$. Then
$P a'$ $(= 300 \times 60 \div 100) = 180$, the distance of a' from P.

To find $a' c$—

$$\text{Formula.} \quad a' c = \frac{a' c' \times a d'}{a d} \text{ (p. 24).}$$

Taking the distances as above, and $a' c' = 90$. We find—

$$a' c \; (= 90 \times 60 \div 40) = 135, \text{ the distance of } c \text{ from } a'.$$

To find the inaccessible distance $a\,a'$—

Formula. $a\,a'\left(\dfrac{=m\,a \times P\,d'}{m\,d'}\right)=\dfrac{m\,a \times P'\,d}{m\,d'}$ (p. 45).

Let $m\,a=290$, $m\,d'=300$, and $P\,d'=240$. Then,

$a\,a'\,(=290 \times 240 \div 300)=232$, the inaccessible distance.

3. Reference point. Split line. Perpendicular and area of triangle (fig. 4).

The formula are—

$m=\dfrac{a^2+b^2-c^2}{2\,a}$, $n=(l^2+m^2-b^2)^{\frac{1}{2}}$, and $l=(b^2-m^2)^{\frac{1}{2}}$ (p. 26).

In these formula—

Let $a=500$, $b=400$, $c=700$, and $l=282{\cdot}3$. Then,

$m\,(=(250000+160000-490000)\div 1400)=285{\cdot}70$ nearly,

$n\,(=(80871{\cdot}+81629{\cdot}-160000)^{\frac{1}{2}})=50$. And

$m+n\,(=285{\cdot}7+50)=335{\cdot}7$, the distance of the reference point d from the angular point C.

Let n be given $=50$; to find l—

$l\,(=(160000+2500-81629)^{\frac{1}{2}})=282{\cdot}3$:

To find the perpendicular—

$p\;=(l^2-n^2)^{\frac{1}{2}}$. Then

$p\,(=(80871-2500)^{\frac{1}{2}})=279{\cdot}8$ nearly.

And to find the area A—

$A\left(=\dfrac{p \times a}{2}\right)=\dfrac{279{\cdot}8 \times 500}{2}$, or $A=69950$ nearly.

4. Inaccessible distances (fig. 14):—

For this method the formula are—

$m\,n=\dfrac{b\,m \times m\,m'}{p\,b'}$, or $m\,n=\dfrac{a\,m \times m\,m'}{p'\,a'}$, or

$m\,n=\dfrac{v_1\,1 \times c\,1}{c\,3}-m\,v_1$ (p. 42).

In these formula let the given distances be—

$b\,m=240$, $m\,m'=111$, $b\,b'=200$, $a\,m=110$, and $a\,a'=151{\cdot}8$

From these the following are found by obvious reductions :—

$p\,b' = 89$, and $p\,a' = 40{\cdot}8$. Then

$$\left.\begin{array}{l} m\,n\ (=240 \times 111 \div 89) \quad = 299{\cdot}327 \\ \text{Or } m\,n\ (=110 \times 111 \div 40{\cdot}8) = 299{\cdot}327 \end{array}\right\}\text{the inaccessible distance.}$$

Again, let $v_1\,1 = 115$, $v_2\,2 = 152{\cdot}27$, $v_3\,3 = 196{\cdot}33$, $v_1\,v_2 = 110$ and $v_1\,v_3\ (= c\,1) = 240$.

From these the following are found by obvious reductions :—

$c\,3 = 81{\cdot}33$ and $c'\,2 = 37{\cdot}27$. Then

$$\left.\begin{array}{l} v_1\,n\ (=115 \times 240 \div 81{\cdot}33) = 339{\cdot}5 \\ \text{or } v_1\,n\ (=115 \times 110 \div 37{\cdot}27) = 339{\cdot}4 \end{array}\right\}\text{ the distance } v_1\,n.$$

And $m\,n\ (= 339{\cdot}4 - 40) = 299{\cdot}4$, the inaccessible distance as above.

5. Inaccessible distances (fig. 15).

Angles measured with a theodolite. The formula is $\cot\,\phi = p$ (p. 44). Supplying the radius r, and taking logarithms, we have—

$\log\cot\phi = \log p + 10$. And when

			°	′	″
$p=1$, $\log 1 + 10\ (=10{\cdot}0000000) = \log\cot\phi$	then	$\phi=$	45	0	0
$p=2$, $\log 2 + 10\ (=10{\cdot}3010300) = \log\cot\phi$	,,	$\phi=$	26	3	54
$p=3$, $\log 3 + 10\ (=10{\cdot}4771212) = \log\cot\phi$	,,	$\phi=$	18	26	6
$p=4$, $\log 4 + 10\ (=10{\cdot}6020600) = \log\cot\phi$	,,	$\phi=$	14	2	$10\frac{1}{2}$
$p=5$, $\log 5 + 10\ (=10{\cdot}6989700) = \log\cot\phi$	,,	$\phi=$	11	18	35·8
$p=6$, $\log 6 + 10\ (=10{\cdot}7781513) = \log\cot\phi$	,,	$\phi=$	9	27	44·28

6. Spherical excess. Spherical triangles, area, &c. The formula for 'spherical excess' (e) is—

$$\tan\tfrac{1}{4}e = \sqrt{\tan\tfrac{1}{2}s\,\tan\tfrac{1}{2}(s-a)\,\tan\tfrac{1}{2}(s-b)\,\tan\tfrac{1}{2}(s-c)}^{*}\ (\text{p. } 63).$$

Let the sides of a spherical triangle be expressed in degrees, and let

$a = 4^\circ\ 30'\ 0''$, $b = 2^\circ\ 30'\ 0''$, and $c = 3^\circ\ 0'\ 0''$. Then $s = 5^\circ\ 0'\ 0''$
$\frac{1}{2}(s-a) = 0^\circ\ 15'\ 0''$, $\frac{1}{2}(s-b) = 1^\circ\ 15'\ 0''$, $\frac{1}{2}(s-c) = 1^\circ\ 0'\ 0''$.

---

* Thomson's Trigonometry, ed. 2 p. 56.

By substituting these values in the formula, supplying the radius r, and taking logarithms, we have:—

log tan $\frac{1}{2}s$	=	8·6400931
log tan $(s-a)$	=	7·6398201
log tan $(s-b)$	=	8·3388563
log tan $(s-c)$	=	8·2419215
		2)32·8606910
$\frac{1}{2}$ sum − 10	=	6·4303455
(= log tan $\frac{1}{4}e$)		
e (= 0° 0′ 55″ × 4) = 0° 3′ 42″·253		

log tan 0° 0′ 55″	=	6·4259376
log tan $\frac{1}{4}e$	=	6·4303455
diff. logs.	=	——44079
diff. logs for 1″	=	——78253

Then $\frac{44079}{78253} = 0''{\cdot}563$. And

°	′	″
0	0	55
+0	0	0·563
0	0	55·563 = $\frac{1}{4}e$

To find the area (A):—

From page 62 the spherical excess, $e = \dfrac{A \times 180}{\pi r}$.

From Geometry we have $\pi r = \dfrac{2\pi \times 2r}{4}$, which is an expression for the fourth part of the surface of a sphere; but πr is the area of a great circle, and hence the area of a sphere is four times the area of a great circle. And from the formula we have $A = \dfrac{\pi r \times e}{180}$, e being expressed in degrees.

Let the diameter of the earth, for mean surface, be 7,916 miles, hence r=3958.

Roughly let $r \times 3{\cdot}1416^* = \pi$. Then

$$\pi r = r^2 \times 3{\cdot}1416, \text{ and } A = \frac{r^2 \times 3{\cdot}1416 \times e}{180}.$$

2 × log r	=	7·1949516
log e	=	$\bar{2}$·7905455
log 3·1416	=	0·4971509
		5·4826470
log 180	=	2·2552725
log A	=	3·1273745
log 1340·8	=	——3640 nearest tab. log.
diff. logs	=	——105 corresponding to 0·04

e	=	0° 3′ 42·252″
Or e	=	0° 3′·7042
Or e	=	0°·061737 nearly.

∴ A (= 1340·8 + 0·04) = 1340·84 square miles.

* It would be more correct to take 3·1415926536 for 3·1416.

7. To find the difference of altitude from vertical angles (fig. 21).

For this purpose the formula are:—

$$Bn \cot e = BC, \text{ and } \frac{CB \cos}{\cos \phi} = AC \text{ (p. 66).}$$

Let $Bn = 30$ miles, $e = 0° \; 26' \; 03''\cdot 42$, $\phi - 10° \; 20' \; 35''\cdot 2$, and $\theta = 10° \; 5' \; 20''$. Then taking logarithms:—

$\log Bn = 1\cdot 4771212$	$\log \cot 0° \; 26' \; 0'' = 12\cdot 1212923$
$\log \cot e$	diff. for $1'' = $ ——2793
$-10 = 2\cdot 1203371$	diff. for $3''\cdot 42 = $ ——9552
$\log BC = 3\cdot 5974583 = 3957\cdot 9$	$\log \cot 0° \; 26' \; 3''.42 \;\; 12\cdot 1203371$

8. To estimate the approximate error due to inter-atmospheric refraction we have $r = \frac{1}{2}\,(CAb + Z - DBa)$ (fig. 22, p. 67). In this formula $CAb = \theta$ and $DBa = \phi$. By substitution $r = \frac{1}{2}(10° \; 5' \; 20'' + 0° \; 26' \; 3''\cdot 42 - 10° \; 20' \; 35''\cdot 2)$, or $r = 0° \; 5' \; 24''\cdot 11$. Correcting for refraction we have $\theta' \; (= \theta - r) = 9° \; 59' \; 55''\cdot 89$, and $\phi' \; (= \phi + r) = 10° \; 25' \; 59''\cdot 31$. Substituting these corrected values in the formula for finding AC, and taking logarithms, we have:—

$\log BC$	$= 3\cdot 5974583$	
$\log \cos \theta'$	$= 9\cdot 9933531$	
	$13\cdot 5908114$	
$-\log \cos \phi'$	$= 9\cdot 9927598$	
$\log AC$	$= 3\cdot 5980516$	
log. $3963\cdot 3$	$=$ ——69	
diff. logs	$=$ ——53	
Corres. diff. of numbers	$= 0\cdot 5$	
Then		
$AC \; (= 3963\cdot 3 - 0\cdot 5)$	$= 3962\cdot 8$	
From $3962\cdot 8 = AC$.		
Take $3957\cdot 9 = BC$.		
Diff.	$4\cdot 9 = $ alt. A above B.	

	°	′	″	
log cos	10	0	0	$= 9\cdot 9933515$
$\theta' =$	9	59	55 9	
diff. $=$	0	0	04·1	
diff. logs per $1'' = 3\cdot 8$				
$\therefore$ diff. log per $4''\cdot 1$			$= +$	——16
$\log \cos \theta'$				$= 9\cdot 9933531$
	°	′	″	
log cos	10	26	0	$= 9.9927595$
$\phi' =$	10	25	59·31	
diff. $=$	0	0	0·69	
diff. log per $1'' = 3\cdot 9$				
$\therefore$ diff. logs per $0\cdot 69''$			$= +$	——3
$\log \cos \phi$				$= 9\cdot 9927598$

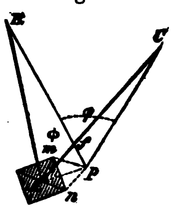
Here the angle in the table is greater than the given angle: its log cosine will, therefore, be less than that of the given angle. Hence the sign $(+)$ to the difference of the log cosines.

9. Finding the angles at an inaccessible trig. point (fig. 24). The formula are $\theta - \phi = \frac{r \sin(\phi + y -)}{a} - \frac{r \sin y}{b}-$, and $\theta - \phi + B p C = A$ (p. 73), the required angle.

In these formula ϕ represents the measured angle.

θ the required angle.

Fig. 24.



y the angle made at the accessible point by lines from the inaccessible point and the left-hand side distant point.

a and b the right and left-hand lines containing the required angles.

Let $B p m = 76° 9' 16''$, $B p C = 54° 30' 10''$, and $m p n = 48° 0' 0''$; and let $m n = 60$ and $m p = 80$, also $a = 44960$ and $b = 48980$ approximately. Then, taking logarithms :—

log mp	=	1·9030900			° ′ ″
log sin mpn	=	9·8710735	mpn	=	48 0 0
		11·7741635	mnp	=	82 14 50
$-$ log mn	=	1·7781512			130 14 50
log sin 82° 14′ 50″ =		9·9960123			180 0 0
(= mnp)			nmp	=	49 45 10
			Amn	=	45 0 0
			Amp	=	94 45 10

In the triangle $A m p$, $A m \left(= \frac{(m n^2)^{\frac{1}{2}}}{2}\right) = 42·426$. And $\frac{1}{2}(A m + m p) = 61·213$, also $\frac{1}{2}(A m - m p) = 18·787$. To find the other sides and angles, when the two sides and the contained angle are given, we have :—

$\frac{1}{2}(180^\circ\ 0'\ 0'' - A\,m\,p) = 42^\circ\ 37'\ 25''$.

$\log \frac{1}{2}(A\,m - m\,p) = 1{\cdot}2738574$
$\log \tan \frac{1}{2}(\pi - A\,m\,p)$
$= 9{\cdot}9639332$
$11{\cdot}2377906$
$- \log \frac{1}{2}(A\,m + m\,p)$
$= 1{\cdot}7868437$

Then

log tan	9.4509469
log tan $15^\circ\ 46'\ 20'' =$	——-384
diff. logs =	85

diff. logs per $1'' = 80{\cdot}5$. And corresponding angular diff. $= 1''$.

$15^\circ\ 46'\ 20''$
$= \quad\quad 1$

log tan $\frac{1}{2}$ diff. of the base angles $= 9{\cdot}4509469$ ($=$ log tan $15\ 46\ 21$)

15°	$46'$	$21''$	half diff. of base angles
42	37	25	half sum of base angles
58	23	46	the greater base angle
26	51	04	the lesser base angle.

Again—

$\log m\,p = 1{\cdot}9030900$
$\log \cos\ 4^\circ\ 45'\ 10'' = 9{\cdot}9985041$
($=$ log sin 94 45 10)
$11{\cdot}9015941$
$-$log sin 58 23 46 $= 9{\cdot}9302822$
$\log A\,p\ (= r) = 1{\cdot}9713119 = (93{\cdot}607)$.

log sin $58^\circ\ 23'\ 40''$ $\cdot 9302745$
diff. logs for $1'' = 12{\cdot}9$
$\therefore$ diff. logs for $6'' = +$ ——77
log sin $58^\circ\ 23'\ 46'' = 9{\cdot}9302822$

Now

$y\ (= 76^\circ\ 9'\ 16'' + 26^\circ\ 51'\ 4'')$	$=$	103°	$0'$	$20''$
ϕ	$=$	54	30	10
$(\phi + y)$	$=$	157	30	30

And therefore

$(\theta - \phi)$ may be computed as follows:—

$\log r = 1{\cdot}9713119$
$\log \sin (\phi + y) = 9{\cdot}5826871$
($=$ log sin $22'\ 29''{\cdot}30$)
$11{\cdot}5539990$
$-\log a = 4{\cdot}6528263$
$\log \sin A\,C\,p = 6{\cdot}9011727$
($= 0^\circ\ 2'\ 44''$
$+ \quad 0{\cdot}215$)

log sin $0^\circ\ 2'\ 44'' = 6{\cdot}9004187$
do. $\ 0\ \ 2\ \ 45 = 6{\cdot}9034588$
diff. logs per $1'' =$ ——30401
log sin $A\,C\,p$ $\quad 6{\cdot}9011727$
log sin $A\,C\,p$ $-$ log sin $0^\circ\ 2'\ 44''$ $=$ ——7535
$30405 : 1'' :: 7535 : 0{\cdot}215$

$A\,C\,p = 0\ \ 2\ \ 44{\cdot}215$. To find the other remote angle subtended by r;—

log r	= 1·9713119	7·2700073	
log sin y	= 9·9887142	7·2699058	log sin 0° 6′ 24″
	11·9600261	1015	diff. log sines
− log b	= 4·6900188	1295	do. for 1″ therefore
log sin ABp (= 0° 6′ 24″·09)	= 7·2700073	$\frac{1015}{1295}=0{\cdot}09$	increase on angle
		0° 6′ 24″ + 0° 0′ 0″·09	
		= 0 6 24·09,	the angle ABp.

From these we have:—

$$
\begin{array}{rcll}
 & & \;\;\circ\;\;\; \prime\;\;\; \prime\prime & \\
\phi & = & 54\;\;30\;\;10 & \\
+ACp & = & 0\;\;\;2\;\;44{\cdot}215 & \left(=\frac{r\sin\phi+y}{a}\right) \\
\hline
 & & 54\;\;32\;\;54{\cdot}215 & \\
-ABp & = & 0\;\;\;6\;\;24{\cdot}09 & \left(=\frac{r\sin y}{b}\right) \\
\hline
\text{The required angle } A\,(=\theta) & = & 54\;\;26\;\;30{\cdot}125 &
\end{array}
$$

10. Computation of the sides of plane triangles (fig. 26).

In the triangle $A\,B\,a$ let the angles and the sides $A\,B$ be given to find the other sides; *i.e.* let $AB = 400$, $a=44°$, $B=96°$, and $A=40°$. Then by the formula $B\,a=\frac{A\,B\sin A}{\sin a}$ (p. 80). Supplying the radius and taking logarithms—

log AB	= 2·6020600	By natural sines,
log sin A	= 9·8080675	400 × 0·64285 ÷ 0·69478
	12·4101275	= 370·13(= aB)
−log sin a	= 9·8417713	
log aB (=370·13)	= 2·5683562	

11. To compute the angles and side of a plane triangle when two sides and a contained angle are given, let $A\;B\;C$ be the angles of a plane triangle, and $a\,b\,c$ the opposite sides. When the sides $b\;c$ and the angle A are given, we have from trigonometry,

$$\frac{b+c}{2} : \frac{b-c}{2} :: \tan \tfrac{1}{2}(B+C) : \tan \tfrac{1}{2}(B-C).$$

And from this we have—

$$\tan \tfrac{1}{2}(B-C) = \frac{b-c}{b+c} \times \tan \tfrac{1}{2}(B+C).$$

Let $b=400$, $c=370{\cdot}13$, and $A=96°$. Then $(B+C)(=180-96)=84°$, $(b+c)=770{\cdot}13$, $(b-c)=29{\cdot}87$.

Substituting these values in the formula, and taking logarithms—

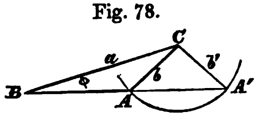
log $(b-c)$	=	1·4752352
log tan $\frac{1}{2}(B+C)$ $(=42°)$	=	9·9544374
		11·4296726
$-$log $(b+c)$	=	2·8865640
log tan $\frac{1}{2}(B-C)$	=	8·5431086
log tan $2°\ 0'\ 0''$	=	——838
diff. logs for 0·4	=	——248
$\frac{1}{2}(B-C)$ = 2 0 0·4		

The angles being known, the remaining side may be computed according to Ex. 10.

	°	′	″	
$\frac{1}{2}(B-C)$ =	2	0	0·4	
$\frac{1}{2}(B+C)$ =	42	0	0	
B =	44	0	0·4	by addition
C =	39	59	59·6	by subtraction
A =	96	0	0	
	180	0	0·0.	

12. To compute for a side and two angles in a plane triangle when the given parts are two sides and an angle opposite one of them. In the diagram (fig 78) let $A\,B\,C$ be the triangle, $a\,b$ the given sides, and B the given angle.

Fig. 78.



Then $\sin A \left(= \dfrac{a \sin B}{b}\right) = \dfrac{a}{b} \times \sin B.$

In the formula, if a be greater than b when B is the given angle, $\frac{a}{b}$ will be greater than 1, and therefore A will be greater than B.

It will be seen by an inspection of the diagram that A and $180-A$, the exterior angle, are both greater than B, an interior angle of the triangle $A\,B\,C$. In this case two triangles, $A\,B\,C$, $A'\,B\,C'$, fulfil the conditions, as the sin of an angle is equal the sin of its supplement.

Let $a=860$, $b=640$, and $B=40°$. Taking logarithms—

$\log a$	$=2{\cdot}9344984$
$-\log b$	$=2{\cdot}8061799$
	$0{\cdot}1283185$
$\log \sin B$	$=9{\cdot}8080675$
$\log \sin A$	$=9{\cdot}9363860$
$\log \sin 59°\ 44'\ 20''$	$=$ ———19
diff. of logs	$41+12{\cdot}3={\cdot}3$

Two angles and two sides being known, the remaining side may be computed from formula in Ex. 10.

		°	′	″
A'	=	59	44	20
+		0	0	0·3
A		59	44	20·3
or		120	15	39·7

13. To compute the angles of a plane triangle when the sides are given. The formula for finding the angle by the tan is—

$$\tan \tfrac{1}{2} A = \left(\frac{(s-b)\times(s-c)}{s\times(s-a)}\right)^{\frac{1}{2}}.$$

And supplying the radius—

$$\frac{\tan \frac{1}{2} A}{r} = \left(\frac{(s-b)\times(s-c)}{s\times(s-a)}\right)^{\frac{1}{2}}.$$

In the formula, $\dfrac{a+b+c}{2}=s$; let $a=860$, $b=640$, and $c=980$. From these we find s, and its excess above each side, as follows—

			$s=1240$
$a=$	860	$(s-a)$	380
$b=$	640	$(s-b)$	600
$c=$	980	$(s-c)$	260
	2)2480		1240
$s=$	1240		

Here the sum of the differences is equal to half the sum of the sides, which verifies the arithmetical operation.

$\log(s-b)$	$=2{\cdot}7781512$	$\log s$	$=3{\cdot}0934217$
$\log(s-c)$	$=2{\cdot}4149733$	$\log(s-a)$	$=2{\cdot}5797836$
$+10$	$=15{\cdot}1931245$		$5{\cdot}6732053$
	$5{\cdot}6732053$		
$\log\tan\frac{1}{2}A$	$=9{\cdot}5199192$		
$\log\tan 18^\circ\ 19'\ 0''$	$=$ ——8819	diff. log per $1''=70{\cdot}6$	
$\cdot 53$	$70{\cdot}6)373(0{\cdot}53$		
$\frac{1}{2}A=18\quad 19\quad 0{\cdot}53$	353		
2	200		
$A=36\quad 38\quad 1{\cdot}16$			

The angle may be also computed by the formula for sines, viz.—

$$\text{Sin } A=\frac{a \sin B}{b} \text{ (p 236).}$$

14. For combined measured bases (fig. 27), the formula is, $C\,C'_n=A\,B\sqrt{3^n}$ (p. 82).

In a particular case let $H\,K=C\,C'_3$. Then—

$$H\,K(=A\,B\sqrt{3^3})=A\,B\sqrt{27}.$$

Let $A\,B=200$. Then, taking logarithms—

log 200	=	2·3010300
$\frac{1}{2}$ log 27	=	0·7156818
log 1039·23 ($=HK$)	=	3·0167118

15. Railway or circular curves :—

a. To find the length of the tangent (fig. 36).

Let the angle made by the tangents $(B)=150^\circ\ 0'\ 0''$ and $R=80$. Then, supplying the radius, the formula for the tangent is:

$$\frac{R}{r}\cot\tfrac{1}{2}B=\text{B}\,t_e \text{ (p. 110).}\quad \text{Taking logarithms—}$$

$\log R$	=	1·9030900
$\log\cot\frac{1}{2}$B	=	9·4280525
$\log B t$ +10	=	11·3311425
$\log B t_e$	=	1·3311425 (= 21·436), the length of the tangent.

b. To find v the secant point in the curve, the formula are :—

$$(\operatorname{cosec} \tfrac{1}{2} B - 1)\, R = B\,v, \quad \text{or}$$
$$(\tan^2 \tfrac{1}{2}(180 - B) + 1)^{\frac{1}{2}} = B\,v, \text{ to the radius 1.}$$

Again $(2\,R + B\,v)\,B\,v = B\,t_c^2$, a quadratic equation.

And $B\,v = (B\,t_c^2 + R^2)^{\frac{1}{2}} - R$.

From the foregoing we have $2 \log B t_c = 2{\cdot}6622850$, and $459{\cdot}5 = B\,t_c^2$

And $-B v\ (= 82{\cdot}82 - 80) = 2{\cdot}82$. This gives the distance of v from B.

	$6400 = R^2$	
	68,59·5	82·82($=B\,o$)
	64	
162	459	
	324	
1648	13550	
	13184	
1656	37600	

By the formula for the cosec we have by reduction—

$$\operatorname{cosec} \tfrac{1}{2} B (= B\,o) = \frac{R \times r}{\sin \tfrac{1}{2} B}. \quad \text{Taking logarithms—}$$

$\log R + 10$	$=$	11·9030900
$-\log \sin \frac{1}{2} B$	$=$	9·9849438
$\log\ 82{\cdot}822$	$=$	1·9181462
$-R = 80{\cdot}000$		
$\therefore B v = 2{\cdot}822$, as above.		

c. To find the secondary tangents $v\,d$. The formula is $R \tan \frac{1}{4}(\pi - B) = v\,d$ (p. 111). Taking B as in the foregoing. Supplying the radius and taking logarithms—

$\log \tan \frac{1}{4}(180° - 37'\ 0'')$	$=$	9·1095594	
$\log R$	$=$	1·9030900	
$\log v d + 10$	$=$	11·0126494	
$\log v d$	$=$	1·0126494	($=$ 10·296)

16. For tracing circular curves with the chain (fig. 37).

The formula for finding the perpendicular to the tangent is :—

$$d\,c = \frac{c\,t_c^2}{2\,R} \text{ (p. 111).}$$

Here R is the radius of the curve, and $d\,c$ the required

perpendicular. Let $R = 80$ and $c\,t_a = 2$. Then $d\,c$ $(=2\times 2\div 80\times 2)=2\div 80$. If the unit of measure be a chain, then $d\,c = 0{\cdot}025$ chains, or $2\frac{1}{2}$ links.

Again, the perpendicular to the chord produced may be found from the formula $d'\,c' = \frac{t_a d' \times d\,c}{t_a d}$ (p. 111). In most cases R will be large as compared with $t_a d'$, then $d'\,c' = 2\,d\,c$. The application of the formula is so simple, that it does not require an example to assist in making the computation clear.

17. Tracing circular curves by perpendiculars to the chord (fig. 38). The formula for finding the perpendicular from a given reference point in the chord are :—

$o\,g = r \sin\theta$

or $o\,g = (r^2 - t_e\,g^2)^{\frac{1}{2}}$, the cosine for half the angle at the centre

$t_a\,t_e = 2\,r\cos\theta$, the chord for the radius r and angle 2θ.

$m\,c = (r^2 - n\,g^2)^{\frac{1}{2}}$, $\left\{\begin{array}{l}\text{the perpendicular to the diameter}\\ \text{parallel to the chord, which perpendi-}\\ \text{cular shall pass through the given}\\ \text{reference point in the chord.}\end{array}\right.$

And $m\,c - o\,g = n\,c$, the required perpendicular (p. 112).

It may be seen that $o\,g$ and $t_a\,t_e$ are computed only once for each curve, and that it is necessary to compute $m\,c$ for each point to be determined in the curve.

Let $R=80$ and $\theta - 75°\ 30'$. Then supplying the radius and taking logarithms—

$\log R$	= 1·9030900	From	$o\,v\ (=80{\cdot}000)$
$\log\sin\theta$	= 9·9849438	Take	$o\,g\ (=77{\cdot}274)$
$\log o\,g + 10$	= 11·8880338	Rem.	$g\,v = 2{\cdot}726$

$\log o\,g\ (=77{\cdot}274) =$ 1·8880338 the max. perpendicular.

To find the chord—

$\log R$	= 1·9030900
$\log\cos\theta$	= 9·4129962
$\log 2$	= 0·3010300
$\log t_a\,t_e + 10$	= 11·5170162
$\log t_a\,t_e$	= 1·5170162 (= 41·411) the chord.

To find the perpendicular at the reference point n—

Let $gn = 10$. Then $(6400-100)^{\frac{1}{2}} = mc$ or $mc = 79{\cdot}372$. And nc $(= 79{\cdot}372 - 77{\cdot}272) = 2{\cdot}125$, the required perpendicular.

18. Tracing circular curves. To find the perpendicular from a given reference point in the tangent. This is solved by Ex. 16 if the corresponding chord be given. By another formula—

$$\left.\begin{array}{ll} R\,(1-\cos\phi) = z\,t_c & (= c'f) \\ \text{And } R\sin\phi \quad = z\,c' & (= t_c f) \end{array}\right\} \text{ (p. 112).}$$

Let the radius of curvature, the angle made by tangents, &c., be taken as above, and let $t_c f = 1{\cdot}396$. Taking the radius 1—

$1{\cdot}396 \div 80$ $(= \sin\phi) = 0{\cdot}01745$. Hence
$\phi = 1°$, and $\cos\phi = 0{\cdot}99985$. Then
$80 \times (1 - 0{\cdot}99985)$ $(= c'f) = 0{\cdot}012$ the required perpendicular.

19 Tracing circular curves by tangents.

To find the reference point in the extreme tangents for a tangent to the curve at given angles from the radius to the point of tangency of the extreme tangents (fig. 40).

The formula are—

$$\begin{aligned} R\tan\tfrac{1}{2}\phi &= q_1, \\ R\tan\phi &= q_2, \\ R\tan\frac{m}{2}\phi &= vd \text{ (p. 114).} \end{aligned}$$

In these formula m is the number of equal subdivisions of the arc, and ϕ the angle at the centre subtended by one of the subdivisions.

Let $R = 80$, $\frac{1}{2}B = 60°$, and $2\,m = 120$. Supplying the radius, and taking logarithms—

$\log\tan\frac{1}{2}\phi$	=	7·9408584	When $\phi = 14°$		
$\log R$	=	1·9030900	$\log\tan\frac{1}{2}\phi$	=	9·0871438
$\log q_1 + 10$	=	9·8439484	$\log R$	=	1·9030900
$\log q_1$ $(= 0{\cdot}698)$	=	1·8439484	$\log q_2 + 10$	=	10·9902338
			$\log q_2 (= 9{\cdot}7776)$	=	0·9902338

The line sq may be computed for an angle $\frac{1}{2}\phi$, as already shown for Bv, Ex. 15.

M 2

20. To find the angle contained between tangents to the curve when the point of intersection of the tangents shall be inaccessible (fig. 43). The formula is—

$$\tfrac{1}{2}\{(A\,m\,f - n\,m\,f -) + (C\,f\,m - n\,f\,m)\} = n \text{ (p. 117).}$$

In this formula let the angles $C\,f\,m = 135°$, and $A\,m\,f = 145°$. Now $180° - 135°$ ($= n\,f\,m$) $= 45°$, and $180° - 145°$ ($= n\,m\,f$) $= 35°$. Substituting these values in the formula we have—

$$\tfrac{1}{2}\{(145° - 35°) + (135° - 45°)\}(= n) = 100°.$$

21. Division of lands (fig. 44).

Let A represent area of the parcel for determination, a the area of the approximate parcel, and $B\,D$ the line indeterminate in position. Then $\dfrac{2(A-a)}{B\,D} = \pm p$ the perpendicular to the intersection of the required boundary lines terminating in $B\,D$ (p. 120).

Let $A = 86972$, a 86201, and $B\,D = 450$. By substitution and reduction $2\left(\dfrac{771}{450}\right) = +p$. Here $p = 3{\cdot}4266$.

22. Proportional division of lands (fig. 45).

Let the parts be a, b, c, d, and let the proportions be—

a	gets	2	proportional parts
b	"	3	" "
c	"	2·5	" "
d	"	1·6	" "
Total of proportional parts		9·1	

Let the total area to be divided be 1697854 square units of area. Then by formula (p. 122)—

$$a = \frac{2}{9{\cdot}1} \times 1697854; \quad a\text{'s portion} = 373154{\cdot}8$$

$$b = \frac{3}{9{\cdot}1} \times 1697854; \quad b\text{'s} \;\;" = 559732{\cdot}2$$

$$c = \frac{2{\cdot}5}{9{\cdot}1} \times 1697854; \quad c\text{'s} \;\;" = 466443{\cdot}5$$

$$d = \frac{1{\cdot}6}{9{\cdot}1} \times 1697854; \quad d\text{'s} \;\;" = 298523{\cdot}5$$

Sum of the areas . . . . 1697854·

23. Proportional division of lands, according to value (fig. 46).

Let the number of parts, the proportionals, and the total area be as in Ex. 22.

Let p_1, p_2, p_3, p_4 be the divisions; and let p_1's proportion be $\frac{2}{9{\cdot}1}$ of the value of the whole area for division; also let the total units of value be £9900. Then the value of p_1's portion is 2175·8243 units of value $\left(=\frac{2\times 9900}{9{\cdot}1}\right)$.

Now let a =697000, and a'=1000854 units of area.

Let v =3982·35 units of value,

and v'=5917·65 „ Farther, let the areas of the approximate parcels c and c' be 200000 and 170000 units of area respectively. We have by substitution in the formula $\frac{v}{a}\times c$ the value of p_{1a}, and $\frac{v'}{a'}\times c'$ the value of $p_{1a'}$, *i.e.*:—

	$\frac{3982{\cdot}35}{697000}\times 200000\ (=p_{1a})$	=	1142·7166	units of value.
And	$\frac{5717{\cdot}65}{1000854}\times 170000\ (=p_{1a'})$	=	971·1611	„ „
	Total units of value of the approximate parcel p_1	=	2113·8777	
	Proportional units of value p_1	=	2175·8243	
	Deficiency in units of value	=	61·9466	

Let mn=13500, $mn_{c'}$=6000, and mn_c=7500 lineal units,

Then 13500 : 61·947 :: 7500 : 34·415 ($=d_{c'}$), and
13500 : 61·947 :: 6000 : 27·532 ($=d_c$).

This gives the deficiency of value on each division of quality. We have $\frac{v}{a}$=quality of a, and $\frac{a}{v}\times 34{\cdot}415$=area on 7500 of mn. Also $\frac{a'}{v'}\times 27{\cdot}532$=area on 6000 of mn.

Taking logarithms—

$\log a$	$=5{\cdot}8432328$	$\log a'$	$=6{\cdot}0003707$
$\log 34{\cdot}415$	$=1{\cdot}5367478$	$\log 27{\cdot}532$	$=1{\cdot}4398378$
	$7{\cdot}3799806$		$7{\cdot}4402085$
$-\log v$	$=3{\cdot}6001395$	$-\log v'$	$=3{\cdot}7721493$
$\log d_a{}^{7500}$	$=3{\cdot}7798511$	$\log d_{a'}{}^{6000}$	$=3{\cdot}6680592$
$-\log \frac{7500}{2}(=3750)$	$=3{\cdot}5740313$	$-\log \frac{6000}{2}(=3000)$	$=3{\cdot}4771212$
$\log$ perp. $(=1{\cdot}606)$	$=0{\cdot}2057198$	$\log$ perp. $(=1{\cdot}552)$	$=0{\cdot}1909380$

These perpendiculars laid down to terminate in the quality line s (fig. 47), determines the intermediate points in the required dividing line. The dividing line should be log spitted from the extremities of the line $m\,m'$ to the corresponding intermediate points in the dividing line, as shown on the diagram referred to. The other proportional divisions p_2, &c., may be determined in like manner.

24. For the intake of lands for railway or like purposes (fig. 48), the formula are—

$h\,m(1)=\frac{n\,b \times h\,c}{n\,c}$, the horizontal surface half width;

$d\,t(2)=(d\,h^2-h\,t^2)^{\frac{1}{2}}$, the horizontal distance for a given distance on a given surface incline;

$t\,r(2)=\frac{n\,c}{n\,b}\times d\,t$, the corresponding vertical height for slope of works and given distance on the given surface incline;

and $h\,m(2)=\frac{c\,h \times h\,d}{r\,h}$, the side width on inclined surface.

The surface, as represented in (3), fig. 48, may be computed in the manner given here for (2) by attending to the formula (pp. 135—6).

Let $n\,b=12$, $n\,c=6$, $h\,n=10$. Then $h\,c(=10+6)$ $=16$, and $\frac{n\,b}{n\,c}\left(=\frac{12}{6}\right)=2$. Also,

$$h\,m\,(1)=2\times16=32, \text{ half width.}$$

Again, as in formula (2)—

Let $h\,t=2$, $d\,h=10$, and $h\,n$, &c., as before. Then, by substitution—

$d\,t\,(=10^2-2^2)^{\frac{1}{2}}=9{\cdot}8$. Substituting this value of $d\,t$ we have—

$t\,r\left(=\frac{6\times 9{\cdot}8}{12}\right)=4{\cdot}9$. And by subtraction—

$h\,r\,(=4{\cdot}9-2)=2{\cdot}9$. Now by substitution we find—

$h\,m\,(2)=55{\cdot}2,\left(=\frac{10\times 16}{2{\cdot}9}\right)$, the half width on the incline.

25. Mining surveys. Capacity or solid content of passages (illust. 59).

	Dist. between sections.	Area of cross section.	Cubic content.
Line 300—Paper area	20	18·92	
Do.	85	20·14	
Sum of consecutive areas		2)39·06	
Paper area	65	19·53 × 65	1269·45
Do.	150	25·44	
Sum of consecutive areas		2)45·58	
Paper area	65	22·79 × 65	1481·35
Do.	230	35·00	
Sum of consecutive areas		2)60·44	
Paper area	80	30·22 × 80	2417·60
Do.	300	30·09	
Sum of consecutive areas		2)65·09	
	70	32·54	2277·80
Cubic content of 280 on line =			7446·20

26. **To find the longitude by lunar distances (fig. 64):—**

At 10 *h.* 15 *m.* 20 *s.* p.m. by clock, Belfast, July 29, 1868, the following measurements were made:—

	°	′	″
Altitude moon's upper limb	70	59	14
Lunar distance to *a* aquilæ	47	48	58
Altitude of *a* aquilæ	42	25	40

Correction on time.

	h.	*m.*	*s.*
Equation of time for time of observation .	0	6	9
Clock too fast	0	8	28
Difference to be subtracted	0	2	19
Time by clock	10	15	20
Mean time of observation	10	13	1

Correction on observed alt. of the moon.

	°	′	″
Moon's semi-diameter	0	15	3
Refraction for altitude	0	0	19
	− 0	15	22
Observed alt. of moon	70	59	14
Correct alt. of the moon's centre . . .	70	43	52
	90	0	0
Corresponding zenith distance	19	16	8

Correction on alt. of *a* aquilæ.

	°	′	″
Refraction for alt. of *a* aquilæ	− 0	1	2·3
Observed alt. do.	42	25	40
Correct alt. of *a* aquilæ	42	24	37·7
	90	0	0
Corresponding zenith distance — *a* aquilæ . . .	47	35	22·3

Correction on observed lunar distance.

	°	′	″
Moon's semi-diameter as above .	0	15	3
Do. lunar distance . . .	47	48	58
Do. correct lunar distance . .	47	33	55

The following are the corrected arcs or angles:—

	° ′ ″	
Correct lunar distance	47 33 55	(a)
Do. a aquilæ zenith do.	47 35 22·3	(c)
Do. moon's do. do.	19 16 8	(b)
From these we have	$2s = 114\ 25\ 25{\cdot}3$	, $a+b+c=2s$
And	$s = 57\ 12\ 42{\cdot}65$	
Also	$s-a = 9\ 38\ 47{\cdot}65$	
	$s-b = 37\ 56\ 34{\cdot}65$	
	$s-c = 9\ 37\ 20{\cdot}65$	
	$s = 57\ 12\ 42{\cdot}65$	

To find the zenith angle A, we have—

$$\tan \tfrac{1}{2} A = \sqrt{\frac{\sin (s-b) \times \sin (s-c) \times r^2}{\sin s \times \sin (s-a)}} \text{ (p. 191).}$$

log sin $(s-b)$	$=9{\cdot}7887880$
log sin $(s-c)$	$=9{\cdot}2231137$
$10-$log sin s	$=0{\cdot}0753700$
$10-$log sin $(s-a)$	$=0{\cdot}7758035$
	$2)19{\cdot}8630752$
log tan $\frac{1}{2}A$	$=9{\cdot}9315376$

log tan 40° 30′ 10″ $=$ ——415

0·9

Too much . 39

$\frac{1}{2} A = 40\ 30\ 9{\cdot}1$

2

diff. logs per $1'' = 42{\cdot}6$

$39 \div 42{\cdot}6 = 0''{\cdot}9$

$A = 81\ \ 0\ 18{\cdot}2$ the zenith angle.

° ′ ″		
37 56 30	=	9·7887754, log diff. per $1'' = 27{\cdot}1$
4·65		126·2, parts for seconds
37 56 34·65	=	9·7887880·2, log sin $(s-b)$
9 37 20	=	9·2231094, log diff. per $1''=124{\cdot}2$
0·35	=	43, parts for seconds
9 37 20·35	=	9·2231137, log sin $(s-c)$
57 12 40	=	9·9246264, log diff. per $1'' = 13{\cdot}6$
2·65		36, parts for seconds
57 12 42·65	=	9·9246300, log sin s
9 38 40	=	9·2241018, log diff. per $1''=123{\cdot}8$
7·65	=	947, parts for seconds
9 38 47·65	=	9·2241965, log sin $(s-a)$

In this calculation the form of the computation gives for $\frac{r^2}{\sin s \sin (s-a)}, \frac{r}{\sin s} \times \frac{r}{\sin (s-a)}$. The arithmetical complement, or log r − log sin s, &c., is taken directly by subtracting the log sin from 10. Again

	°	′	″
Moon's horizontal parallax . . .	0	55	8·6
Do. parallax for altitude . .	+0	17	37·7
Do. semi-diam. and ref. (see p. 248)	−0	15	22
Correction on altitude	0	2	15·7
Observed altitude	70	59	14
Correct alt. moon's centre . . .	71	1	29·7
	90	0	0
Corresponding correct zenith distance =	18	58	30·3 (b')
a aquille do. do. . . . . =	47	35	22·3 (c')
Computed zenith angle . . . =	81	0	18·2 (A)

By the aid of an auxiliary arc or angle θ, the correct lunar distance a' may be computed by the following formula:—

$$\tan \theta = \frac{\tan b' \cos A}{r}, \text{ and}$$

$$\cos a' \,(= m' s') = \frac{\cos b' \times \cos (d' - \theta)}{\cos \theta} \quad \text{(p. 191).}$$

$\log \tan b' \,(= 18^\circ\ 58'\ 30''\!\cdot\!3) = 9\cdot5363577$
$\log \cos A \,(= 81^\circ\ 0'\ 18''\!\cdot\!2) = 9\cdot1940904$

$\log \tan \theta + 10 = 18\cdot7304481$
$\therefore - \log \tan \theta = 8\cdot7304481$

$\log \tan \,(= 3^\circ\ 4'\ 30''\!\cdot\!0) = \text{——}1397$, diff. log tan per $1'' = 392\cdot9$
7·8 3084, parts for seconds

$\theta \quad = 3\ \ 4\ \ 37\cdot8$

Now $\log \cos b' = 9\cdot9757351$
$\log \cos (d' - \theta) = 9\cdot8531500$

$19\cdot8288851$

$\log \cos \theta = 9\cdot9993734$

and $\log \cos a' = 9\cdot8295117$
$(= 47^\circ\ 31'\ 14''\!\cdot\!24)$

The correct lunar distance for time of observation:—

	° ′ ″	
Lunar distance at Greenwich for 9 p.m.	48 10 20	for the given day
" " 12 night	46 57 20	
Difference in 3 hours	1 12 50	

	° ′ ″
Correct lunar distance at Belfast for time of observation	47 31 14·34
Greenwich lunar distance, from above, at 9 p.m. .	48 10 20
Difference of these lunar distances	0 39 5·76

We have now

$1^\circ\ 12'\ 50'' : 0^\circ\ 39'\ 5''{\cdot}76 :: 3$ hours : 1 *h.* 36 *m.* 37·08 *s.*

The fourth term of the proportion is the lapse of Greenwich time beyond 9 p.m. for the corrected lunar distance of the place and time of observation to obtain at Greenwich.

Then 9*h.* 0*m.* 0*s.* + 1*h.* 36*m.* 37·08*s.* =

h. m. s.	
10 36 37·08	Greenwich mean time for corrected lunar distance.
10 13 01	mean time at place for do.
0 23 36·08	difference, or longitude expressed in time.

The longitude is west as the lunar distance is decreasing. The longitude may be expressed in degrees by allowing 15° for each hour of time. The above difference of mean time converted into degrees gives the longitude of Belfast 5° 54′ 2″ nearly.

27. To find the error of the clock, astronomically (fig. 65).

In lat. 54° 36′ 0″, on July 29, 1868, at 9*h.* 46*m.* 18*s.* a.m., the alt. of the sun's lower limb was observed = 44° 47′ 4″·4.

Time of observation from apparent noon 2*h.* 13*m.* 42*s.*

	° ′ ″	° ′ ″
Sun's declination at noon	18 38 59·5	
Do. diff. for 1*h.* = 0° 0′ 39″·5		
Do. do. for $2\frac{1}{4}$*h.* (= 0 1 20) =	0 1 20	
Sun's apparent dec. at time of observation	18 40 19·5	
	90 0 0	
Complement of sun's dec. (*b*)		= 71 19 40 5
Co. lat. (*c*)		= 35 24 0

Sun's semi-diameter	= 0° 15 47″·5 from *Nautical Almanac* (1868).			
Do. parallax for alt.	= 0 0 6·1	do.		
		° ′ ″ 0 15 53·6		
Refraction for altitude	=	0 0 59*		
			° ′ ″ 0 14 54·6	
Observed alt. sun's lower limb	=		44 47 4·4	
Corrected alt. of the sun's centre	=		45 1 59	
Corresponding zenith distance (a)	=			° ′ ″ 44 58 1·
$a+b+c$	=			151 41 41·5
s	=			75 50 50·75

By reduction—

		° ′ ″
$(s-a)$	=	30 52 49·75
$(s-b)$	=	4 31 10·25
$(s-c)$	=	40 26 50·75
s	=	75 50 50·75

Taking the formula for sin or tan, p. 191, we compute from these data, in the triangle $A\,s\,n$, as follows :—

(Formula for tan).

log sin $(s-b)$	=	8·8965187
log sin $(s-c)$	=	9·8120774
$10 -$ log sin s	=	0·0133859
$10 -$ log sin $(s-a)$	=	0·2896719
		2)19·0116539
log tan $\frac{1}{2}\theta$	=	9·5058269
log tan 17° 46′ 10″	=	7965
		304

diff. logs per 1″ = 72·4 $\therefore \frac{304}{72 \cdot 4} = 4 \cdot 2$

				4·2
$\frac{1}{2}\theta$	=	17	46	14·2
				2
θ	=	35	32	28·4 the hour angle from apparent noon.

This, expressed in time of 15° to an hour, is 2*h*. 22*m*. 10·2*s*.

			h. *m.* *s.*
Then	Mean time of observation	=	9 37 49·8
	Time of observation by clock	=	9 46 18
	Difference (clock too fast)	=	0 8 28·2

* See Appendix.

28. Latitude and longitude of trig. points (fig. 68).

In this computation the angle which each reference line makes with a fixed line of direction, such as a meridian, should be known. Let θ represent this angle, $c\,z$ the meridian, and $a\,c$ the given line. Then

$$a\,c \cos\theta = \text{lat. of } c.$$

And $$\text{lat } c \times \tan\theta = \text{long. of } c.$$

These formula give the lat. and long. on rectangular co-ordinates passing through an origin, such as z, and the extremities of the line. If the origin be not z, but another point o, the lat. and long. of the point a will be represented by $o\,x_z$ and $o\,y_z$. From an origin, such as o, the lat. from the axis $o\,y$ for each consecutive angular point will be found by deducting from the sum of the previously computed latitudes for θ in the first and fourth quadrants the sum of like lats. for θ, in the second and third quadrants. [The quadrants in this computation should be reckoned from the origin towards the angular point, right or left—with or contrary to the sun—as the case may be. In what follows the origin will be considered as on the left side, and the reckoning with the sun.] The corresponding longitude will be found in like manner by taking the difference between the like sums of longitude in the first and second, and in the third and fourth quadrants.

The following example computations, and an inspection of diagram 68, will assist in making clear the manner and arrangement for obtaining the results in a tabulated form :—

1	2 Logs.	3	4	5	6 +	7 −	8 Logs.	9
cos (θ) = 60°	9·69897		o y = 3000·00				4·59074	150 × 259·81
300 (= a c) -	2·47712		o x = 2100·00				0·30103	÷ 2
lat. (a c) -	2·17609	+150·00	3150·00	1180·92	19486		4·28971	
tan θ − 10 -	0·23856						2·41465	259·81 ×
							3·01326	1180·92
long. (a c) -	2·41465	+259·81	2359·81	259·81	267870			
							5·42791	
cos 108° 0′ 0″	9·48998						5·31674	259·57 ×
								798·88
840 - -	2·92428						0·30103	÷ 2
− 10 lat. -	2·41426	−259·57	2890·43	921·35	103690		5·01571	
							2·90248	798·88 ×
tan 72° − 10	0·48822						2·96342	921·35
long. - -	2·90248	+798·88	3158·69	1058·69	736040		5·86690	
cos 192° -	9·99040						4·99847	684·69 ×
700 - - -	2·84510						0·30103	145·54
								÷ 2
− 10 lat. -	2·83550	−684·69	2205·74	236·66		49822	4·69744	
tan 12° - -	9·32747							
							2·16297	145·54 ×
							2·37412	236·66
− 10 long. -	2·16297	−145·54	3013·15	913·15		34432	4·53709	
cos. 245° -	9·62594						5·07956	236·67 ×
560- - -	2·74818						0·30103	507·52
								÷ 2
− 10 lat. -	2·37412	−236·67	1969·07	000·00		60055	4·77853	
tan 65° − 10 -	0·33132							
long. - -	2·70544	−507·52	2505·63	305·63				
cos 340° - -	9·97298						4·81730	424·74 ×
452- - -	2·65514						0·30103	154·59
								÷ 2
lat. - - -	2·62812	+424·74	2393·81	424·74		32830	4·51627	
tan 20° - -	9·56106							
long. - -	2·18918	−154·59	2351·04	251·04				
cos 271° -	8·24185							
251·08 - -	2·39981						3·04139	4·38 ×
							0·30103	251·04
− 10 lat. -	0·64166							÷ 2
		+ 4·38	2398·19	429·12		550	2·74036	
tan 89° − 10	1·75807						2·39973	251·04 ×
							2·62812	429·12
long. - -	2·39973	−251·04	2100·00	000·00		106623	5·02785	
cos 360° - -								
601·81 - -					1127086	284322		
lat. - - -		+601·81	3000·00	1030·92	284322			
long. - -		000·00	2100·00	000·00	8·42764			
					4			
					1·71056	8A. 1R. 28·4P.		
					40			
					28·42240			

In the tabulated computation, column 1 contains the given data; 2 contains the logarithms of data and of the lat. and long.; 3 contains the lat. and long.; 4 the reduced lat. and long. to a remote origin o; 5 gives the reduced lat. and long. to rectangular axes passing through extreme angular points; 6 and 7 contain + and − products; 8 contains logarithms of factors and products, and 9 the factors whose logs are inserted in column 8. For the purpose of finding the latitude and longitude for map making, or like purposes, columns 1 to 4 inclusive are those required. For the purpose of finding the area of a close, as in traverse surveying, column 4 is unnecessary. Column 9 is not generally introduced in practice, as all the factors are obtainable from columns 3 and 5.

It may be desirable to explain more fully the products in columns 6 and 7. The first entry in columns 6 or 7 is one half the rectangle of the increments or decrements of lat. and long. on the given line, and the second entry is the rectangle of the lesser reduced latitude of the extremities of the line, and the difference of longitude of the extremities. These products should be placed in columns 6 or 7, according as θ shall be less or greater than 180°. The difference of the sums of these columns, for a 'close' traverse will give the area in units of measure. If the unit of measure be a link, 100 of which shall be equal to 66 feet, or $40 \times 5\frac{1}{2}$ yards, 100,000 square links will be equal to one acre. Hence, by pointing off, in square links, five places of figures to the right, we obtain an expression for the area in acres. The decimal part of this expression may be expressed in roods by multiplying by 'four,' the number of roods in an acre. In like manner the decimal parts of the latter, or roods, may be expressed in square perches by multiplying by 'forty,' the number of square

perches in a rood. This reduction is made in the tabulated example.

29. To find the area of a triangle, the lengths of the sides being given.

The formula is:—

$A = \sqrt{s + (s-a) + (s-b) \times (s-c)}$, in which A represents the area, and a, b, c the lengths of the several sides; also $s = \frac{a+b+c}{2}$.

Let $a = 3565$, $b = 3096$, and $c = 4959$ (fig. 6). Then

	$s = 5810$	Logarithms.	
$a = 3565$	$(s-a) = 2245$	3·3512163	54·84600
$b = 3096$	$(s-b) = 2714$	3·4329693	4
$c = 4959$	$(s-c) = 851$	2·9299295	3·38400
$2s = 11620$	$s = 5810$	3·7641761	40
		2)13·4782912	15·36000
	$A = 54·84600$	6·7391456	
		$A =$ 54a. 3r. 15·4p.	

The area of the triangle may be also computed in the formula—

$$\frac{c \times b \text{ nat sin } A}{2} = \text{area.}$$

In the foregoing triangle $A = 45° \; 36' \; 44''$. Then—

$$\text{Area } \left(= 3096 \times 4959 \times \frac{0·71446}{2} \right) = 5484574 \text{ square links.}$$

This expressed in acres, roods, and perches, is 54a. 3r. 15·32p. This area is slightly less than that computed above, as the natural sin was not taken out to a sufficient number of places.

30. Computation of the areas of parcels from the entries on the content field-book.

The area of the surfaces bounded by the reference lines should be computed by Ex. 29; the area, surveyed by perpendiculars, on these lines should be computed as without. These latter areas should be distinguished

according to the side of the reference line on which they may be. The following tabulated computation gives the data and results in a convenient manner :—

Double areas.	Perpendiculars.	Inter-dis-tance.	Perpendiculars.	Double areas.	Double areas.	Perpendiculars.	Inter-dis-tance.	Perpendiculars.	Double areas.
		970					512		
		28	18	504	55814	118	473		
		396	42	16632					
		66	24				2240		
5900	50	118		1584					
6365	95	67			42480	120	354		
		295					1191		
		970					150	10	1500
12265		20	40	800	98294				1500
					12265				19520
				19520	110559				21020
					21020				
					2)44769				
					22384 to be deducted.				

In this tabulated statement the middle column contains the lengths of the line, and the several inter-reference point distances. The columns right and left contain the sum of the consecutive perpendiculars at the reference points for the intermediate distances, and the extreme columns the products or double areas. (See content field-book, illust. 8, for the survey of the parcel area computed above).

31. To find the scale of map, the area of the grounds represented being given.

Let A represent the area of a well-defined parcel computed from a survey, and a the paper area for same parcel on map to a scale s. Let S be the scale of the

map. As the areas of similar figures are to one another in the duplicate ratio of their homologous sides, we have $A : a :: S^2 : s^2$. From this proportion we obtain—

$$S^2 = \frac{A \times s^2}{a}, \text{ and } S = s\left(\frac{A}{a}\right)^{\frac{1}{2}}.$$

In this formula let A=10a. 2r. 10p., and a=5a. 3r. 30·62p., the map area, by measurement with a scale of 1 in 2376 (three chains to an inch). Expressing the area in acres and taking logarithms—

log 3 =	0·4771212	Or log 2376 =	0·3758464
½ log 10·5625 =	0·5118833	½ log 10·5625 =	0·5118833
	0·9890045		3·8877297
−½ log 5·9414 =	0·3869444	−½ log 5·9414 =	0·3869444
log 4 (= S) =	0·6020601	log S (=3168) =	3·5007854
Scale of map four chains to an inch.		Scale of map 1 in 3168.	

32. Paper areas. Casting content (figs. 79 and 80). For this purpose curved or irregular outlines of detail

Fig. 79.

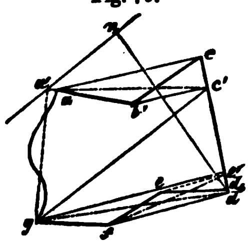
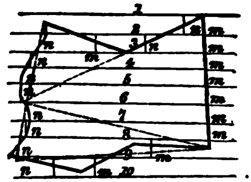


Fig. 80.



on map should be equalised with 'give and take' straight lines, by which an equal area shall be obtained having straight or regular outlines. For very irregular outlines made up of short straight lines and curves, the equalising or 'give and take' line may be conveniently determined by inspection with a transparent

horn or glass straight-edge. On these equalising lines being properly laid down, the computation for area may be conducted in various ways, viz.—

a. The figure may be divided into triangles, more or less well conditioned, and the area of these computed from 'side' × 'half the perpendicular.' [When a side shall be common to two triangles, 'one half the sum of the perpendiculars' × 'side' will give the area of both triangles.] This mode is indicated by dotted lines on fig. 80.

b. The figure may be reduced to an equal one of simpler form with the aid of a parallel ruler, as indicated on fig. 79. The curved part of the irregular outline $a\,b\,c\,d\,e\,f\,g$ should be reduced to a straight line $g\,a'$, as explained without. A quadrilateral equal to this figure, or surface, bounded by straight lines, may be found by drawing a line through b, parallel to $a'\,c$, to intersect $c\,d$ in c'. The triangles $a'\,b\,c$, $a'\,c'\,c$ are equal, and therefore $a'\,c'$ is an equalising or 'give and take' line. The 'give and take' line $f\,e'$, for the part $d\,e\,f$, and also the equalising line $g\,d'$, for the parts $g\,f\,e'$, may be determined in like manner; so that $a'\,c'\,d'\,g$ will be the required quadrilateral. To find the area, $a'\,n$ should be drawn parallel to the diagonal $g\,c'$, and the perpendicular $d'\,n$ let fall from d'. Then

$$\tfrac{1}{2}\,g\,c' \times d'\,n = \text{area}.$$

c. Equidistant parallel lines may be drawn on the map, and the intercepted boundary lines so intersected by perpendiculars that the resulting rectangles shall be equal to the irregular figures bounded by the parallel lines and the intercepted detail lines. The sum of these rectangles will be equal to the area of the parcel; and it is easy to see that the sum of the parallel line

distances between the perpendiculars will express the area. Scales are made and sold for expressing the area, the parallel distance apart being 'fixed' for the purpose. This mode is shown on fig. 80. The methods described require the data to be obtained by measurements with linear scales.

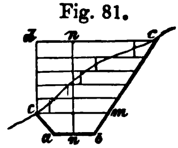
d. Mechanical contrivances—planometers—have been designed and brought into use for integrating the areas of small elements or minute surfaces. These instruments are made chiefly on two different plans. In one, the elements integrated are rectangles; in the other, the integrated elements are triangles. The latter kind of planometers, although not near so large or costly as the former, has almost an equal range. The planometer last alluded to has an index disc-roller placed at the vertex of an isosceles triangle, at which point the hinged equal sides are connected. The centre and outline tracing points are placed at the remote extremities of these equal sides of the variable isosceles triangle. In using the angular planometer, the paper should be laid flat on a plane board or table, that the disc roller may not slip in passing over it, and thus introduce errors into the indicated results. The reading of the index gives the paper area in square inches. The scale of the map being known, the area is easily found by multiplying the area of one square inch into the number of indicated square inches. The other planometer alluded to is made to traverse parallel rails perpendicular to a sliding bar, which carries the tracing point, and gives rotatory motion to a prepared disc on the travelling table of the instrument. The index disc-roller bears on this prepared disc and records its angular and direct motion. The movements of the discs in this class are independent of the nature or form of the surface along which the tracing point shall be con-

ducted. The absolute paper area is also that measured by this instrument.

The accuracy of the indicated area will be increased, and blunders in reading the index detected by repeating the measurements. A mean of several readings free from blunders will give the area with a considerable degree of accuracy.

33. Railway intakes. Computation of areas (fig. 81). The mode for finding the surface half widths has been already explained in Ex. 24. The corresponding horizontal half widths may be found by proportion from the same data as the surface half widths. These half widths are obtained at each peg on the centre line. The pegs are generally placed at equal distances apart, and therefore the common multiplier for the areas between the lines at each consecutive pair of pegs facilitates the finding of the total area. It is easy to see that by multiplying the common peg distance by the sum of the horizontal widths at the intermediate pegs plus half the sum of the widths at the extreme pegs, the product will be the total area. This may be expressed as follows:—Let m = sum of the intermediate widths, and n half the sum of the end widths; also let c be the common distance of the pegs apart. Then $(m+n) \times c$ = area. This is a very simple formula.

Fig. 81.



On sidelong ground, the arbitrary peg point is very frequently ill conditioned for laying off the widths. In those cases, it will also frequently occur that the reference points for side widths on opposite sides of the centre line will not coincide. Hence it may be seen that the half, or side widths, for both sides at each peg point may not be those for a proper intake, or the correct area of

the ground required. The above computation is subject to this serious objection on sidelong ground.

Let *a b* (fig. 81) be formation or grade level, *n n* the centre line, *b c*, *a c* the side slopes, in a cross section of the works. It may be seen, from what is given below, that the side widths from the centre line for each contour plane is constant, *i.e.*, the side widths from the centre *n n* to *c* or *m* is constant for the same inclination of side slopes and contour plane. Hence, if the district within the limits of deviation be contoured, and lines for the side widths to each particular contour be traced or ranged parallel to the centre line, the intersection of the corresponding contour will give the true point on the surface for the half width at that point. If the equidistant contour planes be arranged to give in full detail the features of the surface configuration, the line traced through the points so determined will be the actual surface line of works. This mode of determining the side line of works is correct in every case.

34. For finding the cubic content or quantity of earthworks, the equidistant contour map supplies full data. It may be readily seen by a little consideration of the form and extent of each solid section bounded by the contour planes, surface, and side slopes, &c., and an inspection of fig. 81, that if half the sum of the top and bottom included contour planes *n c*, *a b* be added to the sum of the areas of the like intermediate planes, and this sum be multiplied by the common distance apart of these planes, the product will be the solid content. The formula for finding the solid and surface content when the multiplier shall be a submultiple of the whole depth or distance included, may be demonstrated as follows :— If *a*, fig. 81, represent the area or width of the top contour plane or horizontal line, and *b*, *c*, *d*, &c., *g* the

corresponding successive areas or widths, limited by the side slope $b\,c$ and surface $c\,c$, then we have—

$\frac{1}{2}(a+b)=a'$; $\frac{1}{2}(b+c)=b'$; $\frac{1}{2}(c+d)=c'$; &c. $\frac{1}{2}(f+g)=f'$.

Now

$$\tfrac{1}{2}a+b+c+d+\&c.+f+\tfrac{1}{2}g=a'+b'+c'+\&c.+f';$$

and

$$\tfrac{1}{2}(a+g)+b+c+d+\&c.+f=a'+b'+c'+\&c.+f'$$

the sum of the areas or widths for computation. Again, if p represent the common distance apart. Then

$$\tfrac{1}{2}(a+b)\times p+\tfrac{1}{2}(b+c)\times p \text{ \&c.}=\text{area or content.}$$

Multiplying by p we have—

$$\tfrac{1}{2}\,ap+\tfrac{1}{2}\,bp+\tfrac{1}{2}\,bp+\tfrac{1}{2}\,cp \text{ \&c.}=A.$$

By reduction this comes to be—

$$\tfrac{1}{2}\,ap+bp+cp \text{ \&c. } \tfrac{1}{2}\,gp'=A,$$

g being the extreme area or width. Taking out the common multiplier p we have—

$$(\tfrac{1}{2}(a+g)+b+c+d \text{ \&c.})p^{*}=\text{content.}$$

And $(a'+b'+c'$ &c.) $p'=A'$ the solid content.

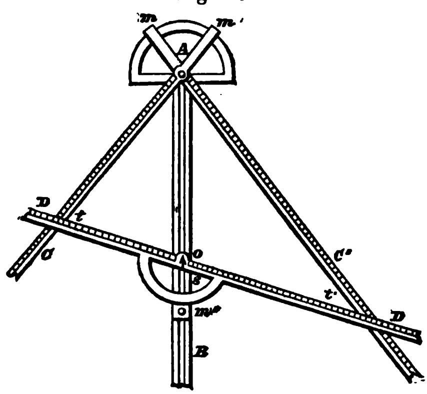
The solid and surface contents for each cutting or filling, and for each section or part of the line, may thus be computed with any required degree of accuracy. If the surface configuration be fully taken up by the equidistant contours, the solid and horizontal surface contents will be accurately obtained by these simple computations. This is the only mode by which the grounds and extent of works can be conveniently and accurately ascertained.

35. Railway intakes. To find the surface distances on grounds for earth-works on side slopes, the area of cross section and approximate solid and surface content, &c.

* This is the well-known Simpsonian formula for surface content. It is believed the application for solid content is new; it has therefore been judged advisable to give the demonstration.

The surface distance $n\,c$ (figs. 81 and 82) from centre to side may be very expeditiously ascertained with the use of the model scale shown in the illustration 82. When the cross section of the ground is a plane, the grounds for the works may be set out with sufficient accuracy for most practical purposes. This will be easily seen from a consideration of the model scale. $A\,B$ is a central bar on which is graduated a scale of equal parts for depths of cutting or height of embankment. The bar $A\,B$ is firmly fixed to a graduated semicircle, having the zero of the divisions in the prolongation of the line $A\,B$, passing

Fig. 82.



through the centre of the graduated circle. A fixed pin in the centre of this circle carries two graduated scale bars $A\,C$, $A\,C'$, which are prolonged to m, m, and each fitted with a clamp screw and index. The lines of graduation of these scales also pass through the centre of the graduated circle. Another bar $D\,D$ is graduated from the centre both ways, and carried on a fixed pin o in a slot

piece, which is fitted free in a groove on the bar $A B$ furnished with a clamp screw m'. This slot piece carries also the index s. A similarly graduated semicircle to that fixed on $A B$ is fixed on $D D$, so as to have its centre in the axis of the pin o. The bar $D D$ may be set to any desired angle by means of the index on the slot piece and the clamp screw m', which clamps at the same time the slot piece and graduated circle to the bar $A B$.

It may be seen by an inspection of the diagram that the graduated straight lines, which have their zeros coinciding in their respective central pins, give the distances in cross section for the several surfaces when the scales or bars are clamped at the respective angles of inclination of those surfaces. If $a b$ represent the grade line, and $t t'$ be the distance on the side slopes from A, we find the area of the cross section of the cutting or embankment from the formula—

$$\tfrac{1}{2}(A t \times A t' - A a \times A b) \times \text{nat. sin } A,$$

In this formula let $A=90°$, and $A a\ (=A b)=20$. Also let $A t=64$, and $A t'=72$. Then, by substitution, $\frac{1}{2}(64 \times 72 - 20^2) \times 1=$area, or $2104=$area of cross section $a b\ t' t$. It may be also seen that $t o$, $t o'$, on the scale $D D$ gives the half-surface widths from the centre line.

CHAPTER VIII.

THE CONSTRUCTION, ADJUSTMENT, AND USE OF INSTRUMENTS REQUIRED IN THE FIELD AND OFFICE WORK OF SURVEYING AND LEVELLING.

THE THEODOLITE.

THE theodolite (illustration 83) is the most generally useful and convenient field angular instrument for surveying purposes. The old and well known make of the theodolite is still generally preferred, although inferior for some purposes, to the small transit theodolite recently introduced. The more recent make of the theodolite differs from the older, in the latter having a complete circle for its graduated vertical limb, as indicated by the name. This limb is so supported, that the telescope may, in turning on the horizontal axis, pass above the horizontal limb, and so be readily reversed without detaching any of the parts. The optical axis of the telescope intersects the horizontal axis line, which gives the measurements without errors due to eccentricity. In the older make the telescope must be detached to be reversed; and it is supported above the horizontal axis on the diametrical bar of a graduated semicircle, having the horizontal axis line passing through its centre. The error due to this eccentricity is so small as to be inappreciable in geodesical operations. In all other respects the older is preferable to the newer make of the

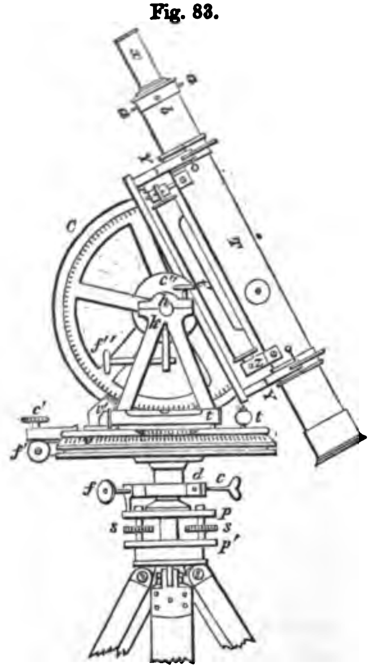
instrument, between which there are not any essential differences. A description of the older make of the theodolite will therefore be sufficient in this place.

The theodolite is made of two essential parts, viz., the horizontal limb for the measurement of horizontal angles, or angles in the horizontal plane, and the vertical limb for the measurement of angles in the vertical plane. The former limb is made to revolve on a vertical, and the latter on a horizontal axis. The necessary mechanical arrangements are, that the axis line shall be truly perpendicular to the plane of its limb, and that the axis line of one limb shall intersect that of the other at right angles. When this arrangement is accurately carried out in the construction, the adjustment which shall place the vertical axis truly vertical shall also place the horizontal axis truly horizontal, the planes of the limbs being perpendicular to one another.

The instrument in which these mechanical arrangements are carried out is placed, for convenience in use, on a strong tripod portable stand capped with a strong plate. This plate has a central screw snug projection on its upper surface, on to which a corresponding socket screw in the centre of the theodolite may be made to work, to properly secure the instrument on the stand. The vertical axis pin projects perpendicularly from the sides of the axis pin plate p, and terminates at one end in the frustum of a cone, and at the other in a ball, having the axis of the cone passing through its centre. A socket in the inside of the screw socket receives the ball. The axis pin passes through a perforation in the socket or lower plate p'. The axis plate has generally four, but sometimes only three threaded sockets made in it near its outer edge. These sockets are equally spaced on the circumference of a circle having its centre in the axis.

Milled head screw pins *s* work into these sockets, and bear on the lower or ball socket plate *p′*. The plates and screws just described are generally called the parallel plates and parallel plate screws. It is not by any means clear how these screws and plates came to be so denominated, since the arrangement is specially designed to destroy the parallelism of the plates. Strictly speaking, the upper of these plates is the vertical axis plate, and the lower the ball-socket, or stand plate. The screws may be denominated the vertical axis plate screws.

Fig. 83.



The horizontal limb is constructed of two circular discs, with their fitting faces accurately ground to plane surfaces. The lower of these discs has a central conical socket secured to it, which socket is ground to fit accurately the conical axis pin of the vertical axis plate, so as to have the same axial line. The ground or fitting surface plane of the horizontal limb plate should be truly perpendicular to this line. In the lower plate, on which the conical socket is cast, or attached, there is

formed a second lesser central socket, having its axis and that of the conical socket in the same straight line. The top plate has a short central axis pin, turned to fit accurately into the lesser socket in the lower plate, in which it is secured by a screw and washer passed through the greater or conical socket. In the English make of the theodolite the outer edge of the lower plate is turned or chamfered to a conical surface—the axis of the cone being in the same straight line with the vertical axis. The chamfered surface is generally silvered, and graduated at its upper or fitting surface edge into degrees and subdivisions of degrees. The outer edge of the upper plate is usually made to stand out, so as to protect the graduations on the lower plate, except where for a short distance, as at *v*, it is chamfered to the same conical surface as the lower plate. On the chamfered surface of the upper plate verniers* are placed, to give a minute subdivision of the graduations on the lower plate. In small theodolites there are usually two such chamfers, &c., at the opposite ends of a diameter. In larger instruments three or more verniers are placed at different parts of the divided circle. Suitable microscopes, carried on arms attached to a slide piece loosely fitted in a groove formed in the outer edge of the lower plate, are usually provided for assisting the observer to read correctly the graduations. A collar and jaws *d* is made to grip the conical axis projection of the lower plate by means of the clamped screw *e*. A slow-motion screw *f* attached to the axis pin plate by a pin and collar is made to work into a screw perforation in a snug on *d*, by which the lower plate may be made to revolve through a small angle after the clamp collar *d* shall be tightened. A

* See 'Vernier,' p. 278.

clamping and slow-motion screw arrangement is shown at $c' f$, by which the upper and lower plates may be clamped and moved through a small angle when so clamped. The arrangement provided for setting the vertical axis truly vertical consists of two level tubes t t attached to the upper plate, at right angles to one another, by screw pin adjustments. These adjustments should be made by placing one of the tubes t in the direction of a diameter through one of the vertical axis plate screws, when the bubble should be brought to the middle of its run by means of the screws s. By turning carefully the limb on its axis through 180°, the level tube will be reversed, and on the opposite side of the axis. It will then be seen if the bubble remains at the middle of its run. If so, the level will be perpendicular to the vertical axis; but if the bubble be not at the middle of its run, it should be moved with the attaching screws one half the distance from the centre and the remainder with the screws s s. The operation should be repeated until the adjustment be accurately made and fully verified. The second level tube t should be attached and adjusted in like manner. These tubes being once properly adjusted should not be afterwards disturbed. This is an instrumental adjustment, and when properly made, the bubbles of the tubes t t should remain at the middle of their respective runs during an entire revolution of the limb on its axis which will then be truly vertical. These are the principal and essential parts and adjustments of the horizontal limb.

The vertical limb is a graduated circle, in whole or in part, mounted on a strong axis bar (perpendicular to its plane) bearing at the ends on supports secured, or attached, to the upper plate of the horizontal limb, so as to make the centre line of the axis bar intersect the vertical

axis line. In the illustration the graduated arc C is a semicircle, h is the end of its axis bar, and K its supports. In the transit theodolite the supports are necessarily longer, and the graduated circle complete; whilst in some theodolites (Everett's) the supports are shorter, and the graduated arcs are parts of the circle at opposite ends of a diameter. The vernier v', for making minute subdivisions of the graduated circle C, is attached to the upper plate of the horizontal limb. A clamp c'' and slow-motion screw f'' are provided for like purposes as those attached to the horizontal limb. These, with a level tube t', to be presently described, are the essential parts of the vertical limb.

To these are added, for greater convenience, accuracy, and powers of observation, a telescope T, and the level tube t'. The position of the telescope is different in different makes of the instruments, but in all makes its centre line intersects the vertical axis. In some it is supported on a strong bar above the horizontal axis bar, as shown in the illustration. In others, the centre of the telescope tube intersects the horizontal axis, as in the 'transit' and in Everett's make. In the latter make the tube t' is usually placed above and on the telescope tube.

In the make of the theodolite illustrated (fig. 83), the telescope is supported in Ys, and secured by hinge pieces attached to the Ys. Strong gun-metal rings are soldered on the tube of the telescope to bear on the Ys. These rings are turned truly and exactly equal, so that the axis of the rings when reversed in the Ys may coincide with the same straight line. The level tube t' for adjusting the axis of the telescope T in the horizontal plane, and the vernier v' to the zero of the graduations on C, is secured and adjusted on snugs attached to the tele-

scope tube. The arrangement shown at r provides for placing the tubes t' and T in horizontal planes. The arrangements shown at z provide for adjusting the axis of these tubes in the same vertical plane.

To make these adjustments, the vertical axis should be adjusted truly vertical, as already described; then the axis of the vertical limb should be examined and found to be truly horizontal. To make this examination, a level, with equal spider legs, should be placed standing on the arbour or turned axis bar of the vertical limb when supported on the stands. If the bubble of the adjusting spider level should stand at the middle of its run, the horizontal axis will be horizontal. The same results should obtain with the adjusting spider level reversed. If this be not the case, the theodolite will be essentially faulty, and not suitable for use. For the adjustment of the tube t' the vertical limb should be clamped when the bubble is near the middle of its run, and the final adjustment of the bubble made by means of the slow-motion screw f''. If, on reversing the telescope in its Ys, the bubble of the level be not at the middle of its run, it should be brought to it, one half by working the adjusting nuts r, and the remainder by working the slow-motion screw f''. The telescope should be carefully re-reversed to verify the accuracy of the adjustment. If on turning the telescope steadily on its axis in its Ys the bubble be observed to move from the middle of its run, the level and telescope tubes will not be in the same vertical plane. The tube t' should be brought into the vertical plane of T by means of the impinging screws at z, the action of which is easily understood.

[In theodolites having the level on the top of a fixed telescope, the lateral adjustment is made by the maker.]

The telescope is made of two principal parts—an outer and an inner tube, F and x, in which are fitted the object-glass and microscope, or eye-piece. To the latter is fitted a diaphragm arrangement $a\,a$, $b\,b$ of platina wire or spider threads. These are called the cross wires, from their being placed crossways and sometimes at right angles to one another. The instrumental adjustment of these parts is confined to the fixing of the intersection of the cross wires in the axis of the equal cylindrical rings bearing on the Ys, or, in other words, in the line of collimation. The optical axis of the object-glass should be made by the maker to coincide with this line. For making the adjustment, the microscope slide-piece should be drawn out and adjusted to obtain distinct vision of the cross wires. Then the telescope (in its Ys) should be directed to a well-defined point of a distant object, and the inner tube moved with the milled head on the side of the telescope so as to obtain distinct vision of the object. When distinct vision of the object and cross wires are obtained, at the same time the focii of the object-glass and microscope coincide. If the cross wires be in the common focii, they will not seem to move on the distant object when the eye is being moved about at the eye end of the telescope. If there be an apparent movement of the cross wires, this *parallax* should be removed by working the milled head on the side of the telescope. Then will the focii of the eye-piece and object-glass be adjusted to the plane of the cross wires. The intersection of the cross wires should now be made, with all the clamps tightened, to cover the well-defined point of the distant object. The telescope should be turned half round in the Ys, and the apparent position of the intersection of the cross wires observed. If the image of the well-defined point remain at the intersection of the

cross wires, the latter will be in the axis of the rings, and if not, not. In the latter case, the intersection of the cross wires should be shifted one half towards the image of the well-defined point by working first the diaphragm screws *a a*, and next *b b*, loosening one screw before tightening the opposite. Then the telescope should be carefully turned on its axis to its first position. If this trial adjustment be accurately made, the intersection of the cross wires will accurately coincide with the image of the well-defined point. If not, the intersection should be made to cover the image of the well-defined point by working the slow-motion screws, and the final adjustment accurately made by proceeding as directed above. Then the image of the well-defined point should appear at the intersection of the cross wires during an entire revolution of the telescope in its Ys.

Before using a theodolite, it is advisable to ascertain if the maker has correctly adjusted the axis of the telescope to be in the vertical plane to which the horizontal axis is perpendicular. This may be ascertained by observing a circumpolar star at its greatest eastern or western elongation, with all the instrumental and field adjustments perfect; then by steadily sweeping down the telescope, without loss of time, and observing the same star by reflection, it will be seen, if the image of the star appear in the latter observation at the intersection of the cross wires, that the maker's adjustment was properly made. If this adjustment be found imperfectly made, the instrument is unsuitable for use. The verniers should be adjusted by the maker.

MEASURING ANGLES WITH A THEODOLITE.—*Angles in a Horizontal Plane.*

A point or object is said to be 'observed' or 'bisected' when the image of the point, or the image of a point in the vertical centre line of the object, shall be at the intersection of the cross wires of the diaphragm. To measure angles accurately, it is necessary that all the instrumental and field adjustments be made very correctly, and that the instrument be free from vibratory motion.

The centre of the instrument or vertical axis should be adjusted in the vertical of the angular point, which may be verified by means of a plummet suspended with a fine silk cord from a hook in the stand under the centre of the instrument, or otherwise by letting a small weight or plummet ball fall freely from under the small hook. The theodolite, when adjusted to the angular point, should be adjusted for observation by placing the vertical axis truly vertical, and drawing out the microscope tube to obtain distinct vision of the cross wires. The former adjustment for observation must be made at each angular point; the latter adjustment will not be disturbed in the field after once made, except by accident, or for packing the instrument in its box. The instrument being properly adjusted for observation, and the clamps all loose, the telescope should be directed to the left-hand object by turning the instrument gently on its vertical axis without disturbing the adjustments. The vertical and horizontal limbs should be clamped, and the object 'observed' by working the slow-motion screws to the vertical limb and the lower plate of the horizontal limb. Then the verniers should be read, and the readings legibly

entered on the field-book. The 'bisection' of the object and the 'reading' should be verified by observation before unclamping the vertical limb or the upper plate of the horizontal limb. After cautiously unclamping these parts or members, the telescope should be directed to a well-defined object having the vertical of its centre in the right-hand line of the angle, by turning the instrument gently on its vertical axis without disturbing the clamp of the lower plate or the other adjustments. The upper plate should be reclamped, and the 'bisection' made by working the slow-motion screw to this plate; then the 'reading' and verification should be made as above directed. The measure of the angle, or the arc passed over by the zero of the vernier, will be found by taking the difference of readings on a vernier, if the reading for the right-hand object be greater than that for the left-hand object. If the reading for the right-hand object be less than the reading for the left-hand object, 360° less by the latter or greater reading must be added to the former to find the measure of the contained angle. A mean of the measured angles thus reduced for each vernier will be more correct than the angle reduced from readings on a single vernier. If it be necessary to obtain the angle with greater accuracy, the errors due to observation and unequal graduations should be removed by measuring the 'arc' or angle several times on different parts of the graduated circle, and taking a mean, first of the measured angle on each arc, reduced from the readings of the verniers as above; and secondly, by taking a mean of the angles so corrected, to obtain the true measure of the angle.

Angles in the Vertical Plane.

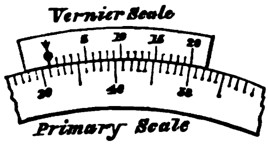
For this purpose the adjustments and observation should be made as above directed, and the vernier to the vertical limb read and entered for elevation or depression, as the case may be. If the observed object be above the level of the theodolite, the angle will be an angle of elevation, but if below that level, it will be an angle of depression. If the vernier reads zero when the bubble of the level tube shall be at the middle of its run, and the line of collimation in the horizontal plane, the reading, for an observed object, will give the vertical angle. If the vernier be not so adjusted as to read zero when the line of collimation shall be adjusted in the horizontal plane, the reading in this case will have an index error, which should be noted and properly applied to the readings for vertical angles to find the correct measure of these angles.

THE VERNIER.

The vernier is a contrivance for reading off more accurately than is practicable with a simple pointer. The graduations or divisions on a line, whether circular or straight, must be of uniform equality to admit of reading with a vernier scale minute subdivisions of a graduation. The graduations of the vernier scale must be also of uniform equality, and so spaced on the graduated circle common to both scales, that an odd number of divisions on the primary scale shall be equal to the next even number and extreme division on the vernier scale. It may be seen by a little consideration, and an inspection of the illustration (fig. 84), that a division of the primary scale will be divided by the zero line of the vernier scale into

as many equal parts as there are divisions in this scale, by the successive coincidence of the graduations on the latter with those of the former scale.

Fig. 84.



This may be stated, generally, as follows:—Let n represent the number of divisions on the vernier scale, then $n-1$ will represent the number on the primary scale equal to n. Let p' and p represent a graduation on the primary and vernier scales respectively; then—

$$n\,p=(n-1)\,p', \text{ and } p'-p=\frac{p'}{n}.$$

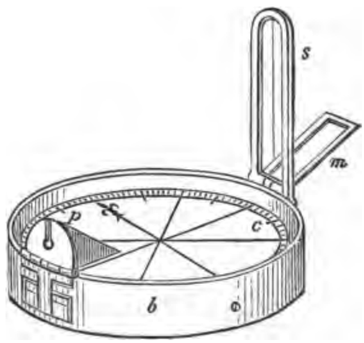
In the formula let $n=20$, then $\frac{p'}{20}=p'-p$. If p' be $\frac{1}{10}$th of an inch, $\frac{1}{20\times 10}\,(=p'-p)=\frac{1}{10}-p$, or $p\left(=\frac{1}{20}-\frac{1}{200}\right)$ $=\frac{19}{200}$ inches, the extent of a division on the vernier scale.

Prismatic Compass.

The prismatic compass is an improvement on the well-known miner's dial. With the improved instrument, the 'bisection' of the object and the graduations on the card, expressing the magnetic angle, may be observed at the same time. The instrument consists of a light brass cylindrical box, open at the top, having a hinged sight vane at one end of a diameter, and a glass prism attached to a sliding piece of brass at the opposite end of the diameter. The illustration (fig. 85) shows the arrangement of the parts of the instrument; s is the sight vane, p the prism, b the box, and c a graduated compass card, securely attached to a magnetic bar,

suitably supported on the fine point of a strong central pin, securely fixed on the bottom of the box; *m* is a hinged plane reflector, revolving on the same axis as *s*, so as to reflect in the diametric plane *sp*. By this arrangement the vertical range of the instrument is considerably increased. There is an eye hole and vane wire slot in the back of the slide piece, to which the

Fig. 85.



prism *p* is securely attached. A fine platina wire is fitted in the sight vane so as to be in a diametric plane perpendicular to the face of the instrument, in which the centre of the eye hole shall also be. In the lower part of the box there is a damper spring for checking the oscillations of the card and supporting it above the pin point, when angles are not being measured with the instrument.

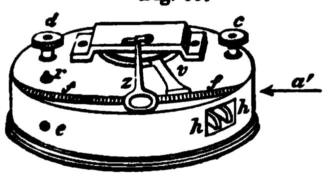
In measuring angles with the prismatic compass, the instrument, held in the hand or supported on a stand, should be so held as to be approximately horizontal, that the diametric sight vane plane shall be approximately in a

vertical plane. In this position the eye should be applied to the eye hole, and the piece p raised or lowered till the graduations on the card and the platina wire shall be in apparent contact. The line of sight being in the diametric plane, objects in any given line bisected by the wire will be in this plane, the magnetic angle of which is expressed by reading the graduation in apparent contact with the wire. The zero of the graduations is adjusted by the maker to be in the magnetic meridian—magnetic vertical plane.

The Box-Sextant.

This instrument, as usually made, is of cylindrical form. It is about three inches in diameter, and one and a half inches in depth. The general external features are shown by the illustration (fig. 86), to which we shall hereafter more particularly refer. It is constructed on the same principle as Hadley's Quadrant, from which it differs only in the construction of its parts. The following brief explanation of the principles of construction will assist in explaining and making the adjustments easily understood.

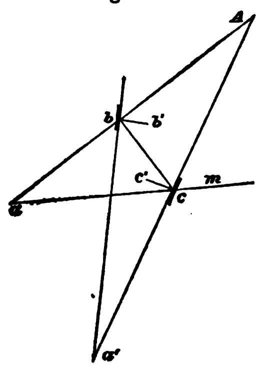
Fig. 86.



It is shown in catoptrics that a ray from A (fig. 87) will be reflected to a along the lines, $A\,b$, $b\,c$, $c\,d$ making the angle $A\,a\,c = 2 \times b\,a'\,c$ in the same plane. The planes of the reflectors $b\,c$, and the axes on which they revolve, should be perpendicular to the plane of reflection. The angles made by the ray on each side of $b\,b'$, or $c\,c'$—the normals to the mirrors b, c,—are equal. These equal

angles, having the normal a common line, are called the angles of incidence ($A\,b\,b'$), and reflection ($b'\,b\,c$).

Fig. 87.



It may be shown from geometry that—

$$A + a' \; (= a'\,b\,c) = a\,b\,a'.$$

And

$$A\,c\,b = a'\,b\,c + a'.$$

By substitution,

$$A\,c\,b \; (= A + 2\,a') = A\,c\,m.$$

Also,

$$A + 2\,a' = A + a.$$

Hence, $a = 2\,a'$.

If the mirror c be fixed perpendicularly to the plane of reflection, and the mirror b be fixed on an axis also perpendicular to and on the same side of that plane, the angle which b shall pass through, as A recedes from m, will be one half the corresponding angle at a. The arrangements and construction provide for measuring on a fixed graduated arc the angle passed over by the index line $b\,a'$ as it revolves on the axis of b to cause the ray $A\,b$ to be reflected from c to a, coincidently with the direct ray $m\,a$, passing through the transparent or unsilvered part of the crystal c. It may be seen by a little consideration that when the observed objects are near to a, a correction for parallax should be made on the measured angle. This parallax is the angle at the object seen by reflection, subtending the line $a\,b$, which for most practical purposes, and distances of about 1,000 feet and upwards, may be disregarded.

The mirrors are usually made of glass, with highly polished parallel faces, silvered on one side. These mirrors

should be set with the planes of their faces perpendicular to the same plane, so that the reflected and direct rays from a distant object may travel in the same plane, usually called the plane of the instrument. To accomplish this, the half-silvered mirror should be fixed perpendicularly to the plane of the instrument. The mirror, which revolves on the axis carrying the index arm *v*, should be set or fixed on the axis bar with a line in its plane parallel to the axis line, so that when the latter shall be adjusted perpendicularly to the plane of the instrument, the face of the former shall be also perpendicular to the same plane. It may be seen that the line of travel of the vernier *v* will be in a plane parallel to the plane of the instrument. The graduated arc *ff* (fig. 86) is made to fit to the travel of the vernier, as scribed from the same centre. The zero of the graduated arc and the zero of the vernier should be so adjusted that they shall coincide when the planes of the mirrors shall be parallel. The parallelism of these planes will be verified if the direct and reflected image of a well-defined remote object, beyond the range of appreciable parallax, be found to coincide. If the sun be the remote object, dark glasses should be interposed by *h h* to moderate the light. For delicacy of movement, the index arm *v* is moved by gearing worked with the milled head *d*. A microscope *z* is carried by a spring bar on *v* to assist in reading the vernier and the graduations on the graduated arc. The numbering of the graduations on the arc *ff* is so made as to give the reading double the actual arc or angle. For some purposes a telescope is added at *a′*.

The instrument may be examined and adjusted as follows:—The direct and reflected images of the opposite limbs of a luminous body, of sensible diameter, such as the sun, or moon at 'full,' should be brought successively

in contact, and the readings of the vernier noted for each. If the readings from zero give the angles equal, the index mirror will be properly set. If these angles be not equal, one half the difference should be corrected with the screw-head *c*. If this be correctly done the readings for each of the opposite contacts will be equal if there be no index error. If in passing from one contact to the opposite the reflected image does not completely cover the direct image, the plane of the fixed glass will not be, as it should be, perpendicular to the plane of reflection. To adjust the fixed glass truly perpendicular to this plane, the key end *r* should be worked so as to make the coincidence of the images perfect. Well-defined terrestrial objects may be selected for these adjustments if they be sufficiently remote to render the parallax inappreciable.

Measuring Angles with a Box-Sextant.

In measuring angles with a box-sextant, the graduated face will be up or down, according as the object seen directly shall be on the left or right-hand side of the observer, looking towards a point between the observed objects. When the axis of two objects are brought into apparent contact, the contained angle may be read off on the graduated arc. As the angle measured with the sextant is in its plane, the range is greater than with the theodolite or the prismatic compass. This increase of range is not, however, of much practical importance, as the position of points are mostly required on horizontal or vertical planes. For the measurement of angles in these planes, the theodolite is much more suitable, as the horizontal plane is not determinable with the sextant. For the measurement of vertical angles, a

reflecting horizon, natural or artificial, is required. For this purpose the instrument should be held so as to observe the elevated object by double reflection, so that the 'direct' ray will be a reflected ray from the reflecting horizon. It is easy to see that the angle measured will be double the angle of altitude. Angles of depression cannot be measured with the sextant.

The optical square is a reflecting instrument similar to the box-sextant, with its mirrors set at an angle of 45° to determine an angle of 90°.

Levelling Instruments.

Levelling instruments are designed to facilitate the accurate determination of points in the horizontal plane of the station of the level telescope axis. The perpendicular to this plane at the levelling instrument will be the resultant of centrifugal force and gravity. As the perpendicular to such resultant line will always be a level line, it follows that the spherical is also the level surface, and hence that lines in the horizontal plane of the levelling instrument are tangents to the level surface at the point. The surface of water in a still pond is an illustration of a level surface, as it is perpendicular to the resultant of centrifugal force and gravity at every point on the surface.

If a glass tube, bent to a curve of large radius, be almost filled with water, or other liquid, and its ends hermetically sealed, the air bubble remaining in the tube will stand at the middle when the ends are placed downwards, if it does not occupy the whole base of the tube. If the ends of the bent tube be supported on a plane surface, so adjusted that a change of the ends of the tube for all points in a circle to a radius of half the distance

between these ends from one point to another on the plane shall not be accompanied by a change in the position of the air bubble, the plane will be horizontal, and coincident with the corresponding level surface at the bent tube point.

In levelling instruments this plane is almost invariably determined as above. Sometimes, as in the case of 'water levels,' the tube is made concave and the ends open. In this case the water surface determines the level line. Reflecting levels are differently adjusted, by the action of centrifugal force and gravity, to have the reflecting surface in the resultant of these forces. These instruments have a very limited range. To increase the range and ensure accuracy in the determinations, an optical contrivance of the telescope class is necessary. This is usually done by adjusting the optical centre line, or line of collimation of a telescope, to the horizontal plane.

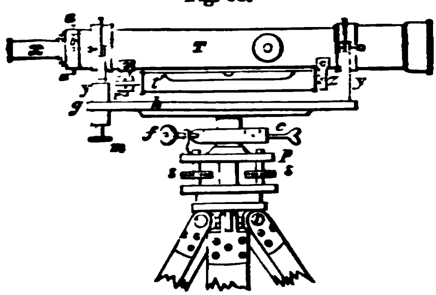
Levelling instruments for the increased range necessarily consists of at least two parts, the one regulated by the combined action of centrifugal force and gravity, the other a combined optical and mechanical arrangement. The latter is somewhat different in different makes of the instrument, which want of sameness is, however, confined to details.

The Y Level.

This level, so named from the form of its bearing supports for the telescope, is a very simple and efficient instrument. The Y level in its simplest form need not be provided with an arrangement for adjusting the axis in a truly vertical position, as in the case of the theodolite, already described. In most instruments of this class, the arrangement of the telescope and its supports are similar

to the like parts in the theodolite, with this difference, that both the Ys in the theodolite are fixed to the horizontal, or diametric bar, whilst in the level only one of the Ys is so fixed. The other Y fits into a socket *g* attached to the eye end of the horizontal bar *h* (fig. 88). A milled head screw works into a corresponding screw in the lower part of *g*, on the end of which screw the socket end of the Y piece bears. The horizontal bar is firmly attached at the middle to a socket, which fits accurately an axis pin attached to the stand-plate. In the more recent make or construction of the instrument, the axis pin is connected with plates and screws, *p s*, similar to those in the theodolite.

Fig. 88.



The adjustment of the line of collimation is essentially the same as for the theodolite, already described. One of the wires, *b*, of the diaphragm will be horizontal when the level tube shall be directly under the telescope. The level tube *t'* may be placed parallel to the axis of *T*,—the line of collimation,—as already described for the theodolite. The adjustment may be also made by using the screw *m*, instead of the parallel plate screws. In the field adjustment of the level, the parallel plate screws, or the

screw *m*, may be used for the purpose of bringing the bubble to the middle of its run. In the former mode the axis should be adjusted truly vertical by placing the telescope successively in the vertical planes of diameters passing through the axis plate screws *s s*, which make right angles with one another. This adjustment will be universal. In the latter mode the adjustment made with the screw *m* must be made independently for every point observed. From this explanation, and an inspection of the illustration, it will appear that it does seem unnecessary to provide two contrivances in the same instrument for effecting the same thing: this has received some consideration from levelling instrument makers.

On the horizontal bar a miner's dial box is placed, with its zero diameter in the vertical plane of the line of collimation. To the vertical axis and socket a clamp and slow-motion screw, *f* and *c*, is attached, with which the accurate bisection of an object in the field of view may be made, to obtain the magnetic angle. The instrument may be used for limited traverse surveys in certain cases.

There is another class of levelling instruments, in which the particular contrivance *m g* is dispensed with.

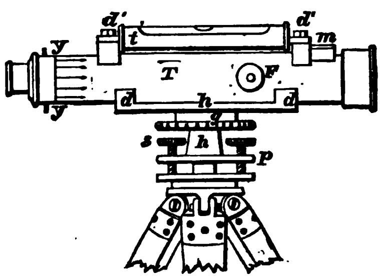
The Dumpey Level.

The Dumpey, Troughton's, and other levels, in which the axis plate and its screws are the only provision for placing the axis vertical and the line of collimation in the horizontal, are improvements on the Y level. The essential parts are a vertical axis, with its plate and plate screws, and a level tube and telescope with a vertical axis socket. In all the makes of the level, up to very recently, the instrument was uselessly encumbered

with a compass box, and card. This is being dispensed with latterly, and the telescope made to bear on the horizontal bar, to which it is attached by a screw adjusting arrangement. In all the makes or forms of this class of level, except Troughton's, in which the level tube is fixed, the screw arrangement for adjusting the principal or direct ray of the object glass perpendicular to the vertical axis is altogether unnecessary, and should be dispensed with, and thus diminish the number of parts liable to get out of adjustment.

The essential adjustments are—the placing of the level tube perpendicular to the vertical axis, so that when the latter shall be truly vertical, the bubble shall remain at the middle of its run during an entire revolution of the instrument on its vertical axis, and the placing of the cross wire of the diaphragm on the image of objects or points in the external horizontal ray. In the illustration (fig. 89) the essential parts are shown at *s*, *T*, *t*, *y*. The

Fig. 89.



necessary instrumental adjustments are made with the screws *d' d'* and *y y*. The field adjustments are made with the screws *s s*. In the present arrangement of the

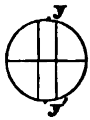
parts the connections at $d\,d$ are unnecessary, as already stated. A similar, but indispensable, connection is shown at $d'\,d'$. This consists of an arrangement of screws under tension and compression, whereby the adjustment of the parts may be delicately and securely made. The arrangement of the adjusting screws at $d'\,d'$ is as follows:—Threaded screw holes are made in the snugs $d'\,d'$, on a centre line parallel to the centre line of the telescope. Corresponding holes are made in a corresponding line in the end snugs of the tube t, but somewhat larger, to allow broad-headed screw pins to be worked into the holes in $d'\,d'$. In each of the end snugs of the level tube a screw hole, on each side of the centre line hole, is made, into which a capstan-headed screw pin works, so as to bear on the snugs $d'\,d'$. This arrangement of the extreme screws under compression, and the intermediate screw under tension, provides for a delicate and secure adjustment of the parts.

In these levels the telescope is fixed, so that for the adjustment or identification of the line of collimation other means must be taken than those for adjusting the line of collimation in the theodolite and Y level. The diaphragm (fig. 90) shows, besides the horizontal wire, two vertical wires, between which the image of the staff should appear on another parallel line. These wires, and those elsewhere described as cross wires, are usually fine platina wire, but sometimes silk or spider thread fibre.* The tube of the telescope is perforated on a vertical diameter for admitting the adjusting screws

* The positions of the cross wires are marked on the face of the diaphragm ring by V-cut scribes. The wire or fibre should be placed in the vertex of the diametric scribes, and fully drawn out before securing it with isinglass, glue, or strong gum.

y y to pass freely and be worked into corresponding screw holes in the diaphragm ring. These screws are for adjusting the horizontal wire to any required point in the vertical line, and, with inside guide pieces, for securing the diaphragm in its place. The position of the diaphragm in the telescope is shown at y y' (fig. 89). The arrangement for adjusting the image of the field object to the focus of the eye-piece, and the latter to the plane of the diaphragm, is similar to that already described for the theodolite in the preceding pages.

Fig. 90.



Adjustment of the Level Tube.

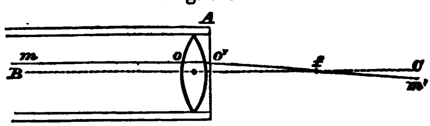
The instrument should be set up as for levelling, and the end snugs of the level tube be made to bear on the seat snugs d' d', attached to the top of the telescope, with all the adjusting screws tightened. The level tube being so seated, and the telescope placed parallel to a screw diameter of the axis plate, the bubble should then be brought to the middle of its run by means of these screws, and the instrument placed again parallel to the same diameter by cautiously turning it on its vertical axis, to reverse its ends. If the bubble now stands at the middle of its run, the level will be perpendicular to the vertical axis. If, however, the bubble does not stand at the middle, it should be brought back one half the distance with the vertical axis plate screws, s s, loosening the one before tightening the opposite, and the remainder with the adjusting screws at d' d'. This adjustment should be also made by placing the telescope parallel to a perpendicular diameter through other screws s s, without disturbing the stand of the instrument, and again repeated

in the former or first position to verify the accuracy with which the adjustment shall be made. When the adjustment shall be accurately made, the bubble will remain fixed at the middle of its run during an entire revolution of the instrument on its vertical axis.

The Telescope.

The telescope is invariably achromatic. It is usually made of two principal parts—an internal and external tube. The eye-piece is placed on the one, and the object-glass on the other. The motion of the internal tube is regulated with a milled head on the side of the telescope, which works gearing inside, as in the telescopes of other instruments. In this form of the levelling instrument the direct or unbent ray passing through the centre of the object-glass, usually called the optical axis, may not be in the centre line of the telescope tube—the line of motion of the inner tube. For the purpose of levelling, it is not necessary, although it is desirable, that these lines should coincide exactly. Until very recently, it was assumed that for the purpose of levelling these lines should coincide; and to this erroneous view of the dioptics of the telescope is due the

Fig. 91.



unnecessary provision, already alluded to, for adjusting the centre line of the telescope perpendicular to the vertical axis. Now it may be seen by a little consideration that the parallel rays *m o* falling on the object-glass *o*

(fig. 91) converge, and pass through the principal focus f on the opposite side of the lens. If one of the converging rays be placed horizontal, the image of all points in this horizontal ray will be always in the parallel part of the path of that ray, on the opposite side of the lens; and if to the parallel path of the horizontal ray the intersection of the cross wires of the diaphragm be adjusted, all points, the image of which shall be at the latter point, will be points in the converging horizontal ray or line. This line will therefore be the line of collimation of the telescope. The line of collimation will not be a single straight line if the intersection of the cross wires be not exactly in the centre line of the object-glass.

In the diagram A represents the object-glass, $B\,C$ the centre or optical axis line, f the principal focus, or focus for parallel rays, $o'\,m'$ a horizontal ray, and $m\,o$ its internal path on the B side of the lens; which is also the direction of motion of the inner tube and the line of collimation, to which the intersection of the cross wires should be adjusted.

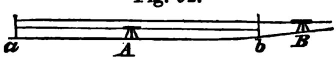
It will depend on the accuracy and perfection of the adjustment of the fixed parts whether the line of collimation shall coincide with the centre line of the lens, or with a line parallel to it.* In all cases the adjustment and verification of the intersection of the cross wires or the horizontal wire to the internal path of the horizontal line, or line of collimation, may be made as follows:—

* It is believed ex-Professor Blood, Queen's College, Galway, was the first in these countries to direct attention to the true determination of the internal path of the outward horizontal ray.

Determination of the Line of Collimation.

The level should be set up on firm ground, sloping about 1 in 120, and accurately adjusted for levelling. Equal distances in a straight line should be measured both ways from the level, and their extremities marked with a hard wood peg driven securely into the ground. These pegs are represented in the diagram (fig. 92) at a b, A being the station of the level; $A\,a$ $(=A\,b)$ may be any convenient length, say about 400 links or 300ft. The

Fig. 92.



level staff, which is a painted scale of equal parts, should be held upright on the peg a, and a reading taken off it from A, taking the top of the horizontal wire, if the latter be not a fine fibre, as the line for reading. Before entering or completing the reading, the eye should be moved about before the eye-piece, to ascertain if there be, or be not, parallax; which, if there be, should be removed, as already explained. In like manner a reading should be taken off the staff held on the peg b without disturbing the level at A, and the verification made as already directed for readings in the measurement of angles. Let A', B', represent these readings, then $A'-B'$ will be the difference of altitude between the points a and b. The level should be removed to a point B, near b, in the straight line a b, and adjusted for levelling as before. From B readings should be taken off the staff held upright on the pegs at b, a, and verified as already explained. Calling these readings b', a', respectively, we will have, when the horizontal wire shall be in the line of

collimation, $A'-B'=a'-(b'+c)$; c being the difference of the earth's sphericity for the distances $B\,d$, $B\,b$. It may be seen by a little consideration that when $A'-B'$ shall be greater than $a'-(b'+c)$, the horizontal wire will be an inclined line, reading below the horizontal; and when $A'-B'$ shall be less, the line will be inclined upwards. Also when $A'-B'$ shall be equal to $a'-b'$, the line of reading will be inclined downwards to compensate for the difference of the earth's curvature for the distances $B\,a$, $B\,b$.

Now let $(A'-B')-(a'-b'-c)=\pm D$. Then by proportion $a\,b : \pm D :: a\,B : \pm D'$. And $a'\pm D'$ will be the reading from the station B, on the staff held upright on the peg at a, to the image of which, with the level undisturbed, the horizontal wire should be adjusted for the identification of the horizontal line, and the determination of points in the horizontal plane. By means of the diaphragm screws $y\,y$ the horizontal wire should be adjusted to make this reading on the staff.

In the adjustment just described there is no attempt made to account for the unnecessary complicated rules and directions which have been advanced in some books on the subject, that are otherwise deservedly of good repute. Allusion is here made to the rules usually given for adjusting the level which generally bears Mr. Gravatt's name, and which has formerly obtained much undeserved favour. The reader who will have made himself acquainted with the simple formula and directions given above, will find himself sufficiently instructed for making the adjustment.

APPENDIX.

TABLE OF REFRACTION FOR THE TEMPERATE ZONES.

Elevation	Refraction			Elevation	Refraction		
	Cassini	Bradley	La Caille		Cassini	Bradley	La Caille
° ′	′ ″	′ ″	′ ″	° ′	′ ″	′ ″	′ ″
0 0	32 20	33 0	33 30	17 30	3 5	2 59	3 18
0 30	30 8	28 22	31 13	18 0	3 0	2 54	3 12
1 0	27 56	24 29	28 57	18 30	2 54	2 49	3 7
1 30	24 30	21 15	25 23	19 0	2 49	2 44	3 3
2 0	21 4	18 35	21 50	19 30	2 44	2 39	2 59
2 30	18 35	16 24	19 15	20 0	2 39	2 35	2 55
3 0	16 6	14 36	16 41	20 30	2 35	2 31	2 51
3 30	14 27	13 6	14 58	21 0	2 31	2 27	2 47
4 0	12 48	11 51	13 16	21 30	2 28	2 24	2 43
4 30	11 40	10 48	12 1	22 0	2 25	2 20	2 40
5 0	10 32	9 54	10 46	22 30	2 21	2 17	2 36
5 30	9 43	9 8	9 44	23 0	2 18	2 14	2 33
6 0	8 55	8 28	8 42	23 30	2 15	2 10	2 30
6 30	8 20	7 51	8 11	24 0	2 12	2 7	2 27
7 0	7 44	7 20	7 41	24 30	2 9	2 5	2 23
7 30	7 15	6 53	7 16	25 0	2 6	2 2	2 20
8 0	6 47	6 29	6 51	25 30	2 3	1 59	2 17
8 30	6 25	6 8	6 30	26 0	2 0	1 56	2 15
9 0	6 4	5 48	6 10	26 30	1 57	1 53	2 12
9 30	5 46	5 31	5 53	27 0	1 55	1 51	2 9
10 0	5 28	5 15	5 37	27 30	1 53	1 49	2 6
10 30	5 13	5 0	5 23	28 0	1 51	1 47	2 4
11 0	4 58	4 47	5 9	28 30	1 49	1 45	2 1
11 30	4 45	4 44	4 57	29 0	1 46	1 43	1 59
12 0	4 32	4 23	4 45	29 30	1 44	1 41	1 56
12 30	4 22	4 13	4 34	30 0	1 42	1 38	1 54
13 0	4 12	4 3	4 24	31 0	1 38	1 35	1 50
13 30	4 3	3 54	4 14	32 0	1 34	1 31	1 46
14 0	3 54	3 45	4 5	33 0	1 30	1 28	1 42
14 30	3 46	3 38	3 57	34 0	1 27	1 24	1 38
15 0	3 38	3 30	3 49	35 0	1 23	1 21	1 35
15 30	3 31	3 24	3 42	36 0	1 20	1 18	1 31
16 0	3 24	3 17	3 35	37 0	1 18	1 16	1 28
16 30	3 17	3 10	3 29	38 0	1 15	1 13	1 25
17 0	3 11	3 4	3 23	39 0	1 12	1 10	1 22

TABLE OF REFRACTIONS, ETC.—continued.

Altitude	Refraction						Altitude	Refraction					
	Cassini		Bradley		La Caille			Cassini		Bradley		La Caille	
°	′	″	′	″	′	″	°	′	″	′	″	′	″
40	1	10	1	9	1	13	65	0	27	0	26	0	31
41	1	7	1	5	1	18	66	0	26	0	25	0	30
42	1	5	1	3	1	14	67	0	25	0	24	0	28
43	1	3	1	1	1	11	68	0	24	0	23	0	27
44	1	1	0	59	1	9	69	0	22	0	22	0	25
45	0	59	0	57	1	6	70	0	21	0	21	0	24
46	0	57	0	55	1	4	71	0	20	0	20	0	23
47	0	54	0	53	1	2	72	0	19	0	18	0	22
48	0	54	0	51	1	0	73	0	18	0	17	0	20
49	0	52	0	49	0	58	74	0	17	0	16	0	19
50	0	50	0	48	0	56	75	0	16	0	15	0	18
51	0	49	0	46	0	54	76	0	14	0	14	0	17
52	0	47	0	44	0	52	77	0	13	0	13	0	15
53	0	45	0	43	0	50	78	0	12	0	12	0	14
54	0	43	0	41	0	48	79	0	11	0	11	0	13
55	0	41	0	40	0	47	80	0	10	0	10	0	12
56	0	40	0	38	0	45	81	0	9	0	9	0	10
57	0	38	0	37	0	43	82	0	8	0	8	0	9
58	0	37	0	36	0	42	83	0	7	0	7	0	8
59	0	35	0	34	0	40	84	0	6	0	6	0	7
60	0	34	0	33	0	38	85	0	5	0	5	0	6
61	0	33	0	32	0	37	86	0	4	0	4	0	5
62	0	31	0	30	0	35	87	0	3	0	3	0	3
63	0	30	0	29	0	34	88	0	2	0	2	0	2
64	0	28	0	28	0	32	89	0	1	0	1	0	1
							90	0	0	0	0	0	0

IMPERIAL STANDARD MEASURES.

MEASURES OF LENGTH.

12 inches	1 foot.
3 feet	1 yard.
$5\frac{1}{2}$ yards	1 pole or rod.
40 poles	1 furlong.
8 furlongs, or 1760 yards . .	1 mile.

Land Measure.

7·92 inches	1 link.
100 links, or 22 yards . . .	1 chain.
80 chains	1 mile.
69·121 miles	1 geo. degree.

Particular Measures of Length.

4 inches	1 hand.
6 feet	1 fathom.
120 fathoms	1 cable's length.
6 points	1 line.
12 lines	1 inch.
3 inches	1 palm.
9 inches	1 span.
144 square inches	1 square foot.
9 square feet	1 square yard.
30¼ square yards	1 square pole.
4 roods, or 4840 sq. yards	1 acre.

Measure of Surface for Land.

62·7264 square inches	1 square link.
10·000 square links	1 sq. chain.
10 square chains	1 acre.
640 acres	1 sq. mile.

Angular Measure; or, Divisions of the Circle.

60 seconds	1 minute.
60 minutes	1 degree.
30 degrees	1 sign.
90 degrees	1 quadrant.
360 degrees, or 12 signs	1 circumference.

Nautical Measure.

1 nautical mile	6082·66 feet.
3 miles	1 league.
20 leagues	1 degree.
360 degrees	the earth's circumference.

French Measures and English Imperial Standard Equivalents.

Measures of Length.

No. of metres.		
0·001 (millimetre)	=	0·0032809 English feet.
0·01 (centimetre)	=	0·0328090 „
0·10 (decimetres)	=	0·3280899 „

No. of metres.			
1 (metre)	=	3·2808992	English feet
10 (decametres)	=	32·808992	„
100 (hectometres)	=	328·08992	„
1,000 (kilometres)	=	3280·8992	„
10,000 (myriometres)	=	32808·992	„

2·539954 centimetres	=	1 inch.
0·914383 metres	=	1 yard.
3·047944 decimetres	=	1 foot.
1·609315 kilometres	=	1 mile.

The standard here is the metre, which bears relation to the circumference of the earth; being the one-forty millionth part of such circumference, or the one-ten millionth part of the distance between the equator and either of the poles.

MEASURE OF SURFACE.

Standard, the 'Are,' being a surface of 100 metres each way.

		In English Acres.
Centiare, or 1-100th part of an 'Are'	=	0·000247
ARE	=	0·024711
Hectare, or 100 'Ares'	=	2·471143

1 square yard = 0·836097 centiares; 1 Acre = 0·404671 hectiare.

The following measures of land formerly obtained in these countries:—

English standard chain	=	66	feet.
Irish plantation chain	=	84	„
Scotch Cunningham chain	=	74·915	„

The areas of similar plane figures are to one another as the square of their corresponding sides. From this we have the areas expressed in different units of surface inversely proportional to the squares, if the sides of the equal figures be expressed in corresponding different units of length. By this proportion, statute measures of surface may be reduced to Irish plantation or Cunningham, and either of these to the other, or the former from the above lengths of the corresponding chains.

Statute Square Measure Reduced to Irish Plantation and Cunningham.

Statute.			Irish Plantation.			Cunningham.		
A.	R.	P.	A.	R.	P.	A.	R.	P.
0	0	1	0	0	·6	0	0	·8
0	0	2	0	0	1·2	0	0	1·5
0	0	3	0	0	1·9	0	0	2·3
0	0	4	0	0	2·5	0	0	3·1
0	0	5	0	0	3·1	0	0	3·9
0	0	6	0	0	3·7	0	0	4·6
0	0	7	0	0	4·3	0	0	5·4
0	0	8	0	0	4·9	0	0	6·2
0	0	9	0	0	5·6	0	0	7·0
0	0	10	0	0	6·2	0	0	7·7
0	0	20	0	0	12·3	0	0	15·5
0	0	30	0	0	18·5	0	0	23·2
0	1	0	0	0	24·7	0	0	31·0
0	2	0	0	1	9·4	0	1	22·0
0	3	0	0	1	34	0	2	13·0
1	0	0	0	2	19	0	3	4
2	0	0	1	0	38	1	2	8
3	0	0	1	3	16	2	1	12
4	0	0	2	1	35	3	0	16
5	0	0	3	0	14	3	3	20
6	0	0	3	2	33	4	2	23
7	0	0	4	1	11	5	1	27
8	0	0	4	3	30	6	0	31
9	0	0	5	2	9	6	3	35
10	0	0	6	0	28	7	2	39
11	0	0	6	3	7	8	2	3
12	0	0	7	1	25	9	1	7
13	0	0	8	0	4	10	0	11
14	0	0	8	2	23	10	3	15
15	0	0	9	1	2	11	2	19
16	0	0	9	3	20	12	1	22
17	0	0	10	1	39	13	0	26
18	0	0	11	0	18	13	3	30
19	0	0	11	2	37	14	2	34
20	0	0	12	1	16	15	1	38
21	0	0	12	3	34	16	1	2
22	0	0	13	2	13	17	0	6
23	0	0	14	0	32	17	3	10
24	0	0	14	3	11	18	2	14
25	0	0	15	1	29	19	1	18
26	0	0	16	0	8	20	0	22
27	0	0	16	2	28	20	3	25
28	0	0	17	1	7	21	2	29
29	0	0	17	3	25	22	1	33
30	0	0	18	2	3	23	0	37
31	0	0	19	0	22	24	0	1
32	0	0	19	3	1	24	3	5
33	0	0	20	1	20	25	2	9
34	0	0	20	3	38	26	1	13
35	0	0	21	2	17	27	0	17
36	0	0	22	0	36	27	3	21
37	0	0	22	3	15	28	2	24
38	0	0	23	1	33	29	1	28
39	0	0	24	0	12	30	0	32
40	0	0	24	2	31	30	3	36
41	0	0	25	1	10	31	3	0
42	0	0	25	3	29	32	2	4
43	0	0	26	2	7	33	1	8
44	0	0	27	0	26	34	0	12
45	0	0	27	3	5	34	3	16
46	0	0	28	1	24	35	2	20
47	0	0	29	0	2	36	1	23
48	0	0	29	2	21	37	0	27
49	0	0	30	1	0	37	3	31
50	0	0	30	3	19	38	2	35
51	0	0	31	1	38	39	1	39
52	0	0	32	0	16	40	1	3
53	0	0	32	2	35	41	0	7
54	0	0	33	1	14	41	3	11
55	0	0	33	3	33	42	2	15
56	0	0	34	2	11	43	1	19
57	0	0	35	0	30	44	0	23
58	0	0	35	3	9	44	3	26
59	0	0	36	1	28	45	2	30
60	0	0	37	0	7	46	1	34
61	0	0	37	2	25	47	0	38
62	0	0	38	1	4	48	0	2
63	0	0	38	3	23	48	3	6
64	0	0	39	2	2	49	2	10
65	0	0	40	0	20	50	1	14
66	0	0	40	2	39	51	0	18
67	0	0	41	1	18	51	3	22
68	0	0	41	3	37	52	2	25
69	0	0	42	2	15	53	1	29
70	0	0	43	0	34	54	0	33
71	0	0	43	3	13	54	3	37
72	0	0	44	1	32	55	3	1
73	0	0	45	0	11	56	2	5
74	0	0	45	2	29	57	1	9
75	0	0	46	1	8	58	0	13
76	0	0	46	3	27	58	3	17
77	0	0	47	2	6	59	2	21
78	0	0	48	0	24	60	1	24
79	0	0	48	3	3	61	0	28
80	0	0	49	1	22	61	3	32
81	0	0	50	0	1	62	2	36
82	0	0	50	2	20	63	2	0
83	0	0	51	0	38	64	1	4
84	0	0	51	3	17	65	0	8
85	0	0	52	1	36	65	3	12
86	0	0	53	0	15	66	2	16
87	0	0	53	2	33	67	1	20
88	0	0	54	1	12	68	0	24
89	0	0	54	3	31	68	3	27
90	0	0	55	2	10	69	2	31
91	0	0	56	0	29	70	1	35
92	0	0	56	3	7	71	0	39
93	0	0	57	1	26	72	0	3
94	0	0	58	0	5	72	3	7
95	0	0	58	2	24	73	2	11
96	0	0	59	1	2	74	1	15
97	0	0	59	3	21	75	0	19
98	0	0	60	2	0	75	3	23
99	0	0	61	0	19	76	2	26
100	0	0	61	2	38	77	1	30
200	0	0	123	1	35	154	3	21
300	0	0	185	0	33	232	1	11
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1000	0	0	617	1	15	774	1	24

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